HG1M12   **Formula Sheet**   Engineering Mathematics 2

Trigonometric functions

$$\tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{\cos x}{\sin x}$$

$$\sec x = \frac{1}{\cos x} \qquad \operatorname{cosec} x = \frac{1}{\sin x}$$

$$\cos^2 x + \sin^2 x = 1 \qquad 1 + \tan^2 x = \sec^2 x$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B \qquad \sin 2A = 2 \sin A \cos A$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B \qquad \cos 2A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

Exponential and logarithm functions

$$\exp(x) = e^x, \quad \ln(e^x) = x, \quad \exp(\ln(x)) = x \ (x > 0)$$

$$e^{a+x} = e^a e^x, \quad \ln(ax) = \ln(a) + \ln(x) \ (a > 0, x > 0), \ \ln x^a = a \ln x \ (x > 0),$$

$$e^0 = 1, \quad \ln(1) = 0, \quad e = 2.7182\ldots$$

Euler's formula

$$e^{\pm i\theta} = \cos\theta \pm i \sin\theta \quad (\theta \text{ in radians})$$

Notation for derivatives

$$u'(x) \equiv \frac{\mathrm{d}u(x)}{\mathrm{d}x}; \ u^{(r)}(x) \equiv \frac{\mathrm{d}^r u(x)}{\mathrm{d}x^r}; \ u_x(x,y) = \frac{\partial u(x,y)}{\partial x}; \ u_y(x,y) = \frac{\partial u(x,y)}{\partial y};$$

$$u_{xy}(x,y) = u_{yx}(x,y) = \frac{\partial^2 u(x,y)}{\partial x \partial y} = \frac{\partial^2 u(x,y)}{\partial y \partial x}$$

Product rule: $\dfrac{\mathrm{d}}{\mathrm{d}x}(fg) = \dfrac{\mathrm{d}f}{\mathrm{d}x}g + \dfrac{\mathrm{d}g}{\mathrm{d}x}f$

Quotient rule: $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{f}{g}\right) = \dfrac{\dfrac{\mathrm{d}f}{\mathrm{d}x}g - \dfrac{\mathrm{d}g}{\mathrm{d}x}f}{g^2}$

Function of a function: If $f(x) = F(u(x))$ then $\dfrac{\mathrm{d}f}{\mathrm{d}x} = \dfrac{\mathrm{d}F}{\mathrm{d}u}\dfrac{\mathrm{d}u}{\mathrm{d}x}$

Methods for integration

Integration by parts: $\displaystyle\int u\frac{\mathrm{d}v}{\mathrm{d}x}\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\mathrm{d}x$

Integration by substitution: $f(x) = F\{u(x)\}\,u'(x); \quad \displaystyle\int f(x)\mathrm{d}x = \int F(u)\mathrm{d}u$

Vector Calculus: The Gradient, Divergence, Curl and Laplacian Operators

If $\phi = \phi(x,y,z)$ is a scalar field and $\mathbf{A}(x,y,z) = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$ is a vector field, where (x,y,z) are cartesian coordinates, then

$$\operatorname{grad}\phi = \nabla\phi = \frac{\partial\phi}{\partial x}\mathbf{i} + \frac{\partial\phi}{\partial y}\mathbf{j} + \frac{\partial\phi}{\partial z}\mathbf{k}, \qquad \operatorname{div}\mathbf{A} = \nabla\cdot\mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z},$$

$$\operatorname{curl}\mathbf{A} = \nabla\times\mathbf{A} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}, \quad \text{and} \quad \operatorname{div}(\operatorname{grad}\phi) = \nabla\cdot(\nabla\phi) = \nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2}$$

Standard derivatives and integrals

$f'(x)$	$f(x)$	$\displaystyle\int f(x)\mathrm{d}x$ (with arbitrary constants of integration omitted)
nx^{n-1}	x^n	$\begin{cases} \dfrac{x^{n+1}}{(n+1)} \ (n \neq -1) \\ \ln|x| \ (n = -1, x \neq 0) \end{cases}$
$k\cos kx$	$\sin kx$	$-\frac{1}{k}\cos kx$
$-k\sin kx$	$\cos kx$	$\frac{1}{k}\sin kx$
$k\sec^2 kx$	$\tan kx$	$-\frac{1}{k}\ln|\cos kx|$
$-k\operatorname{cosec}^2 kx$	$\cot kx$	$\frac{1}{k}\ln|\sin kx|$
ke^{kx}	e^{kx}	$\frac{1}{k}e^{kx}$
$1/x$	$\ln|x|$	$x\ln|x| - x$
$-2x/(a^2+x^2)^2$	$(a^2+x^2)^{-1}$	$\frac{1}{a}\tan^{-1}(x/a)$
$-2x/(x^2-a^2)^2$	$\left(x^2-a^2\right)^{-1}, \ x > a > 0$	$\frac{1}{2a}\ln\left(\dfrac{x-a}{x+a}\right)$
$2x/(a^2-x^2)^2$	$(a^2-x^2)^{-1}, \ |x| < a$	$\frac{1}{2a}\ln\left(\dfrac{a+x}{a-x}\right) = \frac{1}{a}\tanh^{-1}(x/a)$
$-x/(a^2+x^2)^{\frac{3}{2}}$	$(a^2+x^2)^{-\frac{1}{2}}$	$\ln\left\{x + \sqrt{(a^2+x^2)}\right\} = \sinh^{-1}(x/a)$
$x/(a^2-x^2)^{\frac{3}{2}}$	$(a^2-x^2)^{-\frac{1}{2}}, \ |x| < a$	$\sin^{-1}(x/a)$

Taylor Series for functions of one, two and three variables

$$f(x) = f(a) + f'(a)(x-a) + \tfrac{1}{2}f''(a)(x-a)^2 + \tfrac{1}{3!}f'''(a)(x-a)^3 + \cdots$$

$$f(x) = f(0) + xf'(0) + \tfrac{1}{2}f''(0)x^2 + \tfrac{1}{3!}f'''(0)x^3 + \cdots \text{ (Maclaurin series, } a=0)$$

$$f(a+h) = f(a) + f'(a)h + \tfrac{1}{2}f''(a)h^2 + \tfrac{1}{3!}f'''(a)h^3 + \cdots$$

$$f(x,y) = f(a,b) + (x-a)f_x(a,b) + (y-b)f_y(a,b)$$
$$+ \tfrac{1}{2}\left[(x-a)^2 f_{xx}(a,b) + 2(x-a)(y-b)f_{xy}(a,b) + (y-b)^2 f_{yy}(a,b)\right] + \cdots$$

$$f(x,y,z) = f(a,b,c) + (x-a)f_x(a,b,c) + (y-b)f_y(a,b,c) + (z-c)f_z(a,b,c) + \cdots$$

Standard expansions:

$$(1-x)^{-1} = 1 + x + x^2 + x^3 + \cdots \qquad (-1 < x < 1)$$

$$(1+x)^a = 1 + ax + \frac{a(a-1)}{2!}x^2 + \frac{a(a-1)(a-2)}{3!}x^3 + \cdots \qquad (-1 < x < 1)$$

$$\ln(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \cdots \qquad (-1 < x \leq 1)$$

$$e^x = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \cdots$$

$$\sin x = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \cdots$$

$$\cos x = 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \cdots$$

$$\tan x = x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + \cdots \qquad \left(-\frac{\pi}{2} < x < \frac{\pi}{2}\right).$$