

The University of Nottingham

SCHOOL OF MATHEMATICAL SCIENCES

A LEVEL 1 MODULE, SPRING SEMESTER 2018-2019

MATHEMATICAL METHODS FOR ARCHITECTURAL AND ENVIRONMENTAL ENGINEERING

Time allowed TWO Hours THIRTY Minutes

Candidates may complete the front cover of their answer book and sign their desk card but must NOT write anything else until the start of the examination period is announced.

Answer ALL questions

Only silent, self-contained calculators with a Single-Line Display or Dual-Line Display are permitted in this examination.

Dictionaries are not allowed with one exception. Those whose first language is not English may use a standard translation dictionary to translate between that language and English provided that neither language is the subject of this examination. Subject specific translation dictionaries are not permitted.

No electronic devices capable of storing and retrieving text, including electronic dictionaries, may be used.

DO NOT turn examination paper over until instructed to do so

ADDITIONAL MATERIAL: Formula Sheet, Multiple-Choice Answer Sheet, Instructions for Multiple Choice questions

SECTION A

1. If $z = \frac{3 - 2i}{1 + 4i}$ then

(a) $z = \frac{5 - 14i}{17}$

(b) $z = \frac{5 + 14i}{17}$

(c) $\text{Im}(z) = \frac{14i}{17}$

(d) $\text{Re}(z) = -\frac{5}{17}$

(e) None or more than one of the above.

2. If $z = -2 + 2\sqrt{3}i$, then

(a) $z = 4 \left(-\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right)$

(b) $z = -4 \left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3} \right)$

(c) $z = 4e^{i\pi/3}$

(d) $z = 4e^{-2i\pi/3}$

(e) None of the above.

3. If z is a complex number such that

$$3 + 4i - (2 + i)\bar{z} = 0,$$

where \bar{z} is the complex conjugate of z , then

- (a) z is a real number
- (b) z lies in the lower half plane of the Argand diagram
- (c) $|z| = 3$
- (d) $\arg(z) = \pi/6$
- (e) None or more than one of the above.

4. The eigenvalues of

$$\begin{pmatrix} \alpha & \varepsilon & \delta \\ 0 & \beta & 0 \\ 0 & \varepsilon & \gamma \end{pmatrix}$$

are

- (a) $\alpha, \varepsilon, \delta$
- (b) $\alpha, \varepsilon, \beta$
- (c) α, β, γ
- (d) β, γ, δ
- (e) None of the above.

5. Consider the following limit

$$\lim_{x \rightarrow 0} \frac{a - \cos(2x)}{x \sin x}.$$

Which ONE of the following statements is true?

- (a) When $a = 1$ the limit is equal to 2.
- (b) Differentiating the numerator and denominator, using L'Hôpital's rule, we see that the value of a does not affect the value of the limit and so the limiting value is 2, independently of a .
- (c) When $a = 2$ the limit is equal to 1.
- (d) Differentiating the numerator and denominator, using L'Hôpital's rule, we see that the value of a does not affect the value of the limit and so the limiting value is 1, independently of a .
- (e) None of the above statements are true.

6. Let $A(2, 0, 3)$ and $B(5, -1, 8)$ be two points in space. The equation of the plane passing through $C(0, 1, 1)$ and perpendicular to AB is

- (a) $2x + 3z = 3$
- (b) $3x + y + 5z = 6$
- (c) $5x - y + 8z = 9$
- (d) $3x - y + 5z = 4$
- (e) None of the above

7. The function $f(x, y) = (x + y)(xy + 1)$ has

- (a) One local maximum and one local minimum
- (b) Four local maxima
- (c) Two saddle points
- (d) Two minima and one saddle point
- (e) None of the above

8. The Taylor expansion of the function

$$f(x, y) = \frac{x - y}{x + y}$$

about the point $(0, 1)$, up to and including second-order terms, is

- (a) $-1 - 2x + 2x^2 - y^2 - 2x(x - 1) + \dots$
- (b) $-1 + 2x - 2x^2 - 2x(y - 1) + \dots$
- (c) $-1 + 2x + x^2 + \frac{1}{2}x(y - 1) + y^2 + \dots$
- (d) $-1 + 2x + x^2 + y^2 + (x - 1)(y - 1) + \dots$
- (e) None of the above

9. The directional derivative of the vector scalar field

$$\phi(x, y, z) = x^2yz + 4xz^2$$

at the point $(1, -2, -1)$ in the direction $2\mathbf{i} - \mathbf{j} - 2\mathbf{k}$ is

- (a) $17/3$
- (b) 9
- (c) $37/3$
- (d) 21
- (e) None of the above

10. Consider the ordinary differential equation

$$(y^2 - 1)e^x + x^5y^2 + \cos y + [y(Ax^p + 2e^x + y) - x \sin y] \frac{dy}{dx} = 0,$$

where A and p are two real parameters. If this equation is exact, then the value of A is equal to

- (a) 5
- (b) $2/3$
- (c) $5/2$
- (d) $1/3$
- (e) None of the above

SECTION B

11. You are given the following system of simultaneous linear equations in the unknowns x , y and z

$$\begin{cases} 4x + y - 2z = 4 \\ \alpha x - 2y + 8z = 8 \\ -2x - 3y + 10z = 6 \end{cases}$$

for real constant α .

- (a) By writing the system in the matrix form $\mathbf{Ax} = \mathbf{b}$ where $\mathbf{x} = (x, y, z)^T$ is the vector of unknowns, and considering the determinant of \mathbf{A} , determine the only value of the constant α for which there is *not* a unique solution. [4 marks]
- (b) Using the Gauss-Jordan method (row operations) *and no other method*, determine the unique solution to the system when $\alpha = 1$. [4 marks]

12. (a) The matrix \mathbf{B} is given by

$$\mathbf{B} = \begin{pmatrix} 1 & 3 \\ 1 & -1 \end{pmatrix}.$$

Show that $\mathbf{B}^{-1} = \gamma\mathbf{B}$ where γ is a constant you should determine. [3 marks]

- (b) Calculate the eigenvalues and eigenvectors of \mathbf{B} . [5 marks]

13. The function f is defined for real x by

$$f(x) = \frac{2}{1+x^2}.$$

- (a) Calculate the first two non-zero terms of the Taylor expansion for f about $x = 0$ (i.e., the Maclaurin expansion of f). [3 marks]
- (b) Sketch both f and the Taylor expansion of f on the same axes. Indicate all stationary points, asymptotes and locations at which the curves cross the axes. [3 marks]
- (c) Solve the separable differential equation for $y(x)$

$$\frac{dy}{dx} = -xy^2,$$

subject to the boundary condition $y(0) = 2$. [3 marks]

14. Consider the vector field

$$\mathbf{V} = (6x^2y^4 + x)\mathbf{i} + (8x^3y^3 + y)\mathbf{j} + 3\mathbf{k}.$$

i) Show that \mathbf{V} is irrotational.

[2 marks]

ii) Find the most general scalar field $\phi = \phi(x, y, z)$ such that $\mathbf{V} = \nabla\phi$.

[6 marks]

15. A particle of mass $m = 4$ has velocity

$$\mathbf{v}(t) = 12t^2\mathbf{i} + \frac{1}{(1+t)^2}\mathbf{j} + (6t+2)\mathbf{k}, \quad (t \geq 0).$$

i) Determine the position vector of the particle at time $t > 0$ if at the initial time the particle was situated at the point $(1, 1, 0)$.

[3 marks]

ii) Find the acceleration of the particle at time $t > 0$ and hence show that the force $\mathbf{F}(t)$ on the particle at time $t = 1$ is

$$\mathbf{F}(1) = 96\mathbf{i} - \mathbf{j} + 24\mathbf{k}.$$

[3 marks]

iii) Determine the vector moment of the above force about the origin at time $t = 1$.

[3 marks]

16. If $f(x, y) = x^2y + 2xy^2$ and $x = r \cos \theta$, $y = r \sin \theta$, use the chain rule for multivariate functions to show that

$$\frac{\partial f}{\partial r} \cos \theta - \frac{1}{r} \frac{\partial f}{\partial \theta} \sin \theta = r^2(\sin 2\theta - \cos 2\theta + 1).$$

[8 marks]