The University of Nottingham

SCHOOL OF MATHEMATICAL SCIENCES

A LEVEL 1 MODULE, SPRING SEMESTER 2018-2019

ENGINEERING MATHEMATICS 2

Time allowed TWO Hours

Candidates may complete the front cover of their answer book and sign their desk card but must NOT write anything else until the start of the examination period is announced.

Answer ALL Multiple-Choice Questions in Section A. Credit will be given for the best TWO answers in Section B.
Sections A and B are equally weighted.

Only silent, self-contained calculators with a Single-Line Display or Dual-Line Display are permitted in this examination.

Dictionaries are not allowed with one exception. Those whose first language is not English may use a standard translation dictionary to translate between that language and English provided that neither language is the subject of this examination. Subject specific translation dictionaries are not permitted.

No electronic devices capable of storing and retrieving text, including electronic dictionaries, may be used.

DO NOT turn examination paper over until instructed to do so

ADDITIONAL MATERIAL: Formula Sheet, Multiple-Choice Answer Sheet, Instructions for multiple-choice questions

INFORMATION FOR INVIGILATORS: Please collect the multiple-choice answer sheets and scripts separately at the end of the exam

SECTION A

1. The angle between the lines

$$-x = \frac{y}{2} = \frac{z}{3} \quad \text{and} \quad x = 2y = z$$

   is

   (a) $\cos^{-1} \frac{2}{7}$

   (b) $\cos^{-1} \sqrt{\frac{2}{7}}$

   (c) $\sin^{-1} \frac{2}{\sqrt{14}}$

   (d) $\sin^{-1} \frac{1}{7}$

   (e) None of the above

2. The points A and B have coordinates $(6, 2, 1)$ and $(3, -2, 0)$, respectively. The equation of the plane perpendicular to AB and passing through $(2, -1, 0)$ is

   (a) $3x - 2y = 5$

   (b) $6x + 2y + z = 3$

   (c) $3x - 4y - z = -2$

   (d) $3x + 4y + z = 2$

   (e) None of the above

3. Consider the vectors $a = 3\alpha j + 2k$, $b = -i + \alpha^2 j + 5k$ and $c = 3i + k$, for some $\alpha \in \mathbb{R}$. The non-zero value of this parameter that makes these vectors coplanar is

   (a) -8

   (b) 5

   (c) 8

   (d) 48

   (e) None of the above

4. If $f(x, y) = e^{xy}$, then the expression $f_{xx} + f_{yy} + 2f_{xy}$ is equal to

    (a) 0

    (b) $(x + y)^2\, e^{xy}$

    (c) $(x^2 + y^2)\, e^{xy}$

    (d) $(x - y)^2\, e^{xy}$

    (e) None of the above

5. The **curl** of the vector field
$$\mathbf{u} = xz^3\, \mathbf{i} - 2x^2 yz\, \mathbf{j} + 2yz^4\, \mathbf{k}$$
at the point $(1, -1, 1)$ is

    (a) $3\mathbf{j} + 4\mathbf{k}$

    (b) $\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$

    (c) $4\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$

    (d) $2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$

    (e) None of the above

6. A unit normal to the surface
$$(x - 1)^2 + y^2 + (z + 2)^2 = 9$$
at the point $(3, 1, -4)$ is

    (a) $\left(\dfrac{1}{3}, \dfrac{1}{3}, \dfrac{1}{3}\right)$

    (b) $\left(\dfrac{2}{3}, -\dfrac{1}{3}, \dfrac{2}{3}\right)$

    (c) $\left(-\dfrac{2}{3}, \dfrac{1}{3}, \dfrac{2}{3}\right)$

    (d) $\left(\dfrac{2}{3}, \dfrac{1}{3}, -\dfrac{2}{3}\right)$

    (e) None of the above

7. The integrating factor for the first-order linear ordinary differential equation

$$x(x+1)\frac{dy}{dx} - y = 2x^2(x+1), \qquad (x > 0),$$

is

(a) $\dfrac{x}{x+1}$

(b) $x(x+1)$

(c) $\ln x - \ln(x+1)$

(d) $\dfrac{x+1}{x}$

(e) None of the above

8. The solution of the initial-value problem

$$\frac{dy}{dx} = e^{-y}(2x-4), \qquad y(2) = \ln 2,$$

is

(a) $\ln(x^2 - 4x) + 6$

(b) $(x^2 - 4x) + \ln y = \ln 2$

(c) $\ln(x^2 - 4x + 6)$

(d) $\dfrac{\ln x}{2x-4} + \ln 2$

(e) None of the above

9. The function $f(x,y) = -xy\,e^{-(x^2+y^2)/2}$ has

(a) One stationary point

(b) Two stationary points

(c) Four stationary points

(d) No stationary points

(e) None of the above

10. The Taylor series of the function
$$f(x, y) = \ln \frac{x}{\sqrt{x + y}}$$

about the point $(1, 1)$, up to and including second-order terms, is

(a) $-\dfrac{\ln 2}{2} + \dfrac{3}{4}(x - 1) - \dfrac{1}{4}(y - 1) - \dfrac{7}{16}(x - 1)^2 + \dfrac{1}{16}(x - 1)(y - 1) + \dfrac{1}{16}(y - 1)^2 + \ldots$

(b) $-\dfrac{\ln 2}{2} + \dfrac{3}{4}(x - 1) - \dfrac{1}{4}(y - 1) - \dfrac{7}{16}(x - 1)^2 + \dfrac{1}{8}(x - 1)(y - 1) + \dfrac{1}{16}(y - 1)^2 + \ldots$

(c) $-\dfrac{\ln 2}{2} + \dfrac{1}{4}(y - 1) - \dfrac{7}{16}(x - 1)^2 + \dfrac{1}{16}(y - 1)^2 + \ldots$

(d) $-\dfrac{\ln 2}{2} + \dfrac{3}{4}(x - 1) + \dfrac{1}{4}(y - 1) + \dfrac{1}{8}(x - 1)(y - 1) + \ldots$

(e) None of the above

11. A possible function $M(x, y)$ that makes the ordinary differential equation
$$M(x, y) + (\sin x - 4y^3 e^y + x^2)\frac{dy}{dx} = 0$$

an exact equation is

(a) $y \sin x + x^2 y - 4y^3 e^y$

(b) $y^4 e^y + y \cos x + x^2 y$

(c) $3x^2 e^x + y \cos x + 2xy$

(d) $x \sin x - 4xy^3 e^y + x^3$

(e) None of the above

12. The directional derivative of the scalar field
$$\Phi(x, y, z) = 4xz^3 - 3x^2 y^2 z$$

at the point $(2, -1, 2)$ in the direction of the vector $s = (2, -3, 6)$ is

(a) $\dfrac{3}{5}$

(b) $\dfrac{236}{5}$

(c) $\dfrac{376}{7}$

(d) $\dfrac{197}{7}$

(e) None of the above

SECTION B

13. (a) Consider the points

$$A(-1, 0, 2), \qquad B(1, 1, 1) \qquad C(2, -1, -2).$$

    i) Determine the plane $\mathcal{P}$ passing through these three points.   [6 marks]

    ii) Find the coordinates of the intersection point between $\mathcal{P}$ and the line described by the equation

$$\frac{x-2}{3} = \frac{y}{2} = z - 5.$$

[4 marks]

    iii) Calculate the *acute* angle between $\mathcal{P}$ and the plane with Cartesian equation

$$5x + 2y + 7z - 1 = 0.$$

Express your answer in degrees to 4 significant figures.

[5 marks]

(b) The position vector of a particle at time $t \geq 0$ is given by

$$r(t) = (t^2 - 3t + 4)\, i + (4t - 3)\, j - \frac{1}{6}(2t - 3)^3\, k.$$

    i) Determine the velocity vector $v(t)$ and the acceleration vector $a(t)$ of the particle at a generic time t. Is it possible for $v(t)$ to be parallel to the y-axis (of the usual Cartesian system of coordinates)? Justify your answer.

[4 marks]

    ii) Find the acute angle between $v(t)$ and $a(t)$ at time $t = 1$.

[3 marks]

    iii) Calculate the volume of the parallelipiped formed by the three vectors $r(t)$, $v(t)$ and $a(t)$ at $t = 1$.

[3 marks]

14. (a) Consider the vector field F and the scalar ϕ, where

$$F(x, y, z) = 2Axz\,\boldsymbol{i} + 3z^2\,\boldsymbol{j} + (x^2 + 6yz)\,\boldsymbol{k}, \qquad \phi(x, y, z) = xe^z,$$

for some $A \in \mathbb{R}$.
   i) Find the parameter A such that F is irrotational.

   [4 marks]

   ii) Calculate $\nabla\phi$ and $\nabla^2\phi$.

   [4 marks]

   iii) Show by direct calculation that

   $$\nabla \cdot (\phi F) = (\nabla\phi) \cdot F + \phi\,(\nabla \cdot F)\,.$$

   [7 marks]

(b) Show that the ordinary differential equation

$$3x^2 e^y + \sin(x) + \cos(y) + (x^3 e^y - x\sin(y) + 2y)\frac{dy}{dx} = 0,$$

is exact, and then determine its general solution.

   [6 marks]

(c) Find the most general expression of the function $f(x, y)$ which satisfies

$$\frac{\partial^2 f}{\partial y \partial x} = x^3 \sin y + y^3 \cos x\,.$$

   [4 marks]

15. (a) Consider the function

$$f(x, y) = 3x - \frac{3}{4}xy^2 - x^3 .$$

 i) Calculate the partial derivative f_x and f_y.

[2 marks]

 ii) Show that f has four stationary points and classify them.

[12 marks]

(b) If $g(x, y) = 2 + \sin(ye^x)$, then calculate the Taylor series about the point $(0, \pi)$ up to and including linear terms.

[3 marks]

(c) Two variable resistors R_1 and R_2 are connected in parallel so that their combined resistance R is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} .$$

If $R_1 = 100$ ohms $\pm 5\%$ and $R_2 = 25$ ohms $\pm 2\%$, by approximately what percentage can the calculated value of their combined resistance $R = 20$ ohms be in error?

[8 marks]