The University of Nottingham

SCHOOL OF MATHEMATICAL SCIENCES

A LEVEL 1 MODULE, SPRING SEMESTER 2017-2018

ENGINEERING MATHEMATICS 2

Time allowed TWO Hours

Candidates may complete the front cover of their answer book and sign their desk card but must NOT write anything else until the start of the examination period is announced.

Answer ALL Multiple-Choice Questions in Section A. Credit will be given for the best TWO answers in Section B

Marks from sections A and B are equally weighted.

Only silent, self-contained calculators with a Single-Line Display or Dual-Line Display are permitted in this examination.

Dictionaries are not allowed with one exception. Those whose first language is not English may use a standard translation dictionary to translate between that language and English provided that neither language is the subject of this examination. Subject specific translation dictionaries are not permitted.

No electronic devices capable of storing and retrieving text, including electronic dictionaries, may be used.

DO NOT turn examination paper over until instructed to do so

ADDITIONAL MATERIAL: Formula Sheet, Multiple-Choice Answer Sheet, Instructions for multiple-choice questions

INFORMATION FOR INVIGILATORS: Please collect the multiple-choice answer sheets and scripts separately at the end of the exam

SECTION A

1. If $r_1 = 3i - j + 5k$ and $r_2 = i + 2j - k$, then the unit vector in the direction of the vector $r_1 - r_2$ is

   (a) $\dfrac{1}{7}(-2i + 3j - 6k)$

   (b) $\dfrac{1}{7}(2i + 3j + 6k)$

   (c) $\dfrac{1}{7}(2i - 3j + 6k)$

   (d) $-\dfrac{1}{7}(2i - 3j + 6k)$

   (e) None of the above

2. The line passing through the point $(2, 4, -3)$ and perpendicular to the plane

   $$4x + 7y - 4z + 6 = 0$$

   intersects this plane in the point

   (a) $\left(\dfrac{2}{3}, \dfrac{2}{3}, -\dfrac{1}{3}\right)$

   (b) $\left(\dfrac{2}{3}, -\dfrac{2}{3}, \dfrac{1}{3}\right)$

   (c) $\left(-\dfrac{2}{3}, -\dfrac{2}{3}, -\dfrac{1}{3}\right)$

   (d) $\left(\dfrac{1}{3}, -\dfrac{1}{3}, \dfrac{1}{3}\right)$

   (e) None of the above

3. Consider the vectors $u_1 = \lambda i + 3k$, $u_2 = -3i + \lambda j$, $u_3 = 2i - j + k$, for some $\lambda \in \mathbb{R}$. The value of this parameter which makes these three vectors coplanar is

   (a) 5

   (b) -3

   (c) 2

   (d) 3

   (e) None of the above

4. If $f(x, y, z) = y \tan^{-1}(\sqrt{x}) \ln(yz)$, then $\dfrac{\partial^3 f}{\partial x\, \partial y\, \partial z}$ is equal to

    (a) 0

    (b) $\dfrac{y}{2\sqrt{x}\,(1 + x)}$

    (c) $\dfrac{1}{2z\sqrt{x}\,(1 + x)}$

    (d) $\dfrac{\sqrt{x}}{2z\,(1 + x^2)}$

    (e) None of the above

5. The **curl** of the vector field

$$\boldsymbol{u} = z \cos^2(x)\,\boldsymbol{i} + (\sin^2(x) + y)\,\boldsymbol{j} + xyz\,\boldsymbol{k}$$

  is

    (a) $xz\,\boldsymbol{i} - yz\boldsymbol{j} - \cos^2(x)\,\boldsymbol{k}$

    (b) $xz\,\boldsymbol{i} + (\cos^2(x) - yz)\,\boldsymbol{j} + \sin(2x)\,\boldsymbol{k}$

    (c) $z\sin(2x)\boldsymbol{i} + \boldsymbol{j} + xy\,\boldsymbol{k}$

    (d) $z\sin(2x) + xy + 1$

    (e) None of the above

6. The general solution of the separable equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{e}^y \sin(x)$$

  may be written as (with C an arbitrary real constant)

    (a) $y = -\ln|\cos(x) + C|$

    (b) $y = -\ln|\cos(x)| + C$

    (c) $y = -\dfrac{1}{\ln|\cos(x) + C|}$

    (d) $y = \ln|\cos(x) + C|$

    (e) None of the above

7. The integrating factor for the first-order linear ordinary differential equation

$$x^3(x+1)\frac{dy}{dx} - x^2 y = 8\ln^2(x), \qquad (x > 0),$$

is

(a) $\dfrac{x}{x+1}$

(b) $1 + \dfrac{1}{x}$

(c) e^x

(d) $\dfrac{e^{x+1}}{x}$

(e) None of the above

8. The ordinary differential equation

$$2x + 3\alpha\sin(y) - ye^{-x} + (x\cos(y) + \cos(y) + \beta e^{-x})\frac{dy}{dx} = 0$$

is exact if

(a) $\alpha = 3$ and $\beta = 1$

(b) $\alpha = \dfrac{1}{3}$ and $\beta = 1$

(c) $\alpha = \dfrac{1}{3}$ and $\beta = -1$

(d) $\alpha = \beta = 1$

(e) None of the above

9. At the point $(1, -1)$, the function $f(x, y) = 3x^2 - y^2 + 4xy - 2x - 6y$ has

(a) a local minimum

(b) a saddle point

(c) a local maximum

(d) None of the above

10. The Taylor series of the function

$$f(x,y) = \sqrt{x^2 + y^3}$$

about the point $(1,2)$, up to and including first-order terms, is

(a) $3 + \dfrac{x}{3} + 2y$

(b) $3 + \left(\dfrac{x}{3} - 1\right) - 2y$

(c) $3 + \dfrac{1}{3}(x-1) + 2(y-2)$

(d) $3 - \dfrac{1}{3}(x-1) - 2(y-2)$

(e) None of the above

11. A unit normal vector to the surface

$$\sin^2(x) + \cos(2y) - 2z = 0$$

at the point $\left(\dfrac{\pi}{4}, \dfrac{\pi}{4}, \dfrac{1}{4}\right)$ is

(a) $\left(\dfrac{1}{3}, \dfrac{2}{3}, \dfrac{2}{3}\right)$

(b) $\left(\dfrac{1}{3}, -\dfrac{2}{3}, \dfrac{1}{3}\right)$

(c) $\left(\dfrac{1}{3}, -\dfrac{2}{3}, -\dfrac{2}{3}\right)$

(d) $\left(-\dfrac{2}{3}, -\dfrac{2}{3}, -\dfrac{1}{3}\right)$

(e) None of the above

12. The directional derivative of the scalar field

$$\Phi(x,y,z) = (x+y)\ln(x+z) + xyz$$

at the point $(2,3,-1)$ in the direction of the vector $s = (-2,2,1)$ is

(a) -1

(b) $\dfrac{1}{3}$

(c) 1

(d) 0

(e) None of the above

SECTION B

13. (a) Consider the points

$$A\,(1, -2, 3), \qquad B\,(1, 1, 1) \qquad C\,(0, 1, -1)\,.$$

    i) Determine the plane $\mathcal{P}$ passing through these three points.   [6 marks]

   ii) Does $\mathcal{P}$ pass through the point $D\,(0, -2, 1)$? Justify your answer.   [2 marks]

  iii) Calculate the acute angle between $\mathcal{P}$ and the plane with Cartesian equation

$$3y - 4z - 6 = 0.$$

    Express your answer in radians to 5 significant figures.   [5 marks]

(b) A particle of mass 3 has position vector

$$\boldsymbol{r}\,(t) = 4t^3\,\boldsymbol{i} + (1 + 6t^2)\,\boldsymbol{j} + (6t - 5)\,\boldsymbol{k}\,.$$

    i) Determine the velocity vector $\boldsymbol{v}\,(t)$ and the acceleration vector $\boldsymbol{a}\,(t)$ of the particle, and the force $\boldsymbol{F}(t)$ acting on the particle.

                                                                  [6 marks]

   ii) Determine the speed of the particle at time t, simplifying your answer as much as possible.

                                                                   [2 marks]

  iii) Determine the moment about the origin of the force $\boldsymbol{F}(t)$ acting at the point $\boldsymbol{r}\,(t)$.   [2 marks]

  iv) Determine a parametric equation of the straight line passing through the points whose position vectors are $\boldsymbol{r}\,(0)$ and $\boldsymbol{r}\,(1)$, respectively.

                                                                    [2 marks]

14. (a) The critical buckling load P for a beam of rectangular cross section is given by the formula

$$P \equiv P(b, h, L) = K\left(\frac{bh^3}{L^2}\right),$$

where K a material constant, L represents the initial length of the beam, while b and h ($b > h$) characterize the rectangular cross-section.
The percentage errors in measuring b, h and L are $+1\%$, $+2\%$ and -0.5% respectively.

   i) Compute the percentage error in P.

   [10 marks]

   ii) Compute the maximum percentage error in P.

   [2 marks]

(b) Show that the ordinary differential equation

$$\cos(y) + y\cos(x) + (\sin(x) - x\sin(y))\frac{dy}{dx} = 0,$$

is exact, and then find its solution which satisfies $y = -1$ when $x = 0$.

   [6 marks]

(c) If $f(x, y) = x^2 + 2yx^2$ and $x = r\cos\theta$, $y = r\sin\theta$, use the chain rule for multivariate functions to show that

$$f_r \sin\theta + \frac{1}{r}f_\theta \cos\theta = 2x^2.$$

   [7 marks]

15. (a) Find the general solution of the linear ordinary differential equation

$$x(1-x)\frac{dy}{dx} - (x-1)y = 1 - x^2, \quad x > 1,$$

and then determine the particular solution which satisfies $y(1) = 1$.

[6 marks]

(b) Find and classify all stationary points of the function

$$f(x, y) = x^6 + 7y^5 + 96x^2 - 35y + 8.$$

[7 marks]

(c)   i) If $\phi = \phi(x, y, z)$ and $\psi = \psi(x, y, z)$ are two scalar fields, then check by direct calculations that

$$\nabla \cdot (\phi \nabla \psi) = (\nabla \phi) \cdot (\nabla \psi) + \phi \nabla^2 \psi.$$

[8 marks]

ii) Let $f = f(x, y, z)$ and $g = g(x, y, z)$ be two **harmonic** scalar fields. Show that the vector field $f(\nabla g) - g(\nabla f)$ is **solenoidal**.

[4 marks]