University of Nottingham
School of Mathematical Sciences

HG1 M12 Engineering Mathematics 2

Coursework Assignment **Submission deadline: 3pm, 20 March 2018**

This assessed coursework should be submitted to your local Student Services Centre. Please show all your workings, add explanations were possible and make sure that the document is reasonably tidy and readable. Marks are assigned for all the following aspects: Methodology, clarity of exposition, correctness of results. Detailed marking criteria can be consulted on Moodle.

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1. (a) Show that the function

$$u(x,t) = t^{-1/2}\mathrm{e}^{-x^2/4t}$$

satisfies

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}\,.$$

(b) Hence, or otherwise, show that the function

$$v(x,y,t) = t^{-1}\mathrm{e}^{-(x^2+y^2)/4t}$$

satisfies

$$\frac{\partial v}{\partial t} = \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\,.$$

[20 marks]

2. (a) Consider the skew lines

$$\mathcal{L}_1: \quad \mathbf{r} = \mathbf{a}_1 + t\mathbf{d}_1\,, \qquad (t \in \mathbb{R})\,,$$

$$\mathcal{L}_2: \quad \mathbf{r} = \mathbf{a}_2 + s\mathbf{d}_2\,, \qquad (s \in \mathbb{R})\,,$$

where $\mathbf{a}_1 = (-1, 2, -3)$, $\mathbf{a}_2 = (1, -1, 3)$, $\mathbf{d}_1 = (15, -4, -7)$ and $\mathbf{d}_2 = (-3, -10, -4)$.

i. If P and Q are arbitrary points on $\mathcal{L}_1$ and $\mathcal{L}_2$, respectively, then show that PQ^2 is given by the expression

$$f(t,s) \equiv (2 - 15t - 3s)^2 + (3 - 4t + 10s)^2 + (6 + 7t - 4s)^2\,.$$

ii. Show that the function $f(t,s)$ defined above has only one critical point (t_0, s_0), where t_0 and s_0 are values that you must specify. Determine the nature of this critical point.

iii. Consider the point P_0 on $\mathcal{L}_1$ corresponding to the value $t = t_0$, and the point Q_0 situated on $\mathcal{L}_2$ and for which $s = s_0$. Show that the line P_0Q_0 is perpendicular on both $\mathcal{L}_1$ and $\mathcal{L}_2$.

iv. Find the length of P_0Q_0. What does this represent?

(b) i. Show that the equation of the plane passing through the point with position vector **a** and parallel to the vectors **b** and **c** can be expressed in the form

$$[\mathbf{r}, \mathbf{b}, \mathbf{c}] = [\mathbf{a}, \mathbf{b}, \mathbf{c}]\,, \tag{1}$$

where **r** denotes the position vector of an arbitray point (x, y, z) in the aforementioned plane.

ii. If $\mathbf{a} = (1, -1, 3)$, $\mathbf{b} = (0, 5, 7)$ and $\mathbf{c} = (-2, 1, 4)$, write equation (1) in the form $Ax + By + Cz + D = 0$, where A, B, C and D are constants that you must specify.

[35 marks]

3. The orbital period T of a satellite moving in a circular orbit around a spherical planet satisfies the equation

$$\frac{T^2}{R^3} = \frac{4\pi^2}{GM}\,, \tag{2}$$

where R is the distance from the centre of the planet to the satellite, $G = 6.673 \times 10^{-11}$ $N \cdot m^2/Kg^2$ is a physical constant, and M represents the mass of the planet (expressed in Kg).

(a) Use equation (2) to express T as a function of R and M.

(b) Assuming that the percentage change in R and M are 1% and -2%, respectively, calculate the corresponding percentage change in the period of the satellite.

[15 marks]

4. An experimentalist obtains the following data

x	3.0	5.42	7.85	10.28	12.71	15.14	17.57	20.0
y	2.18	3.52	3.51	3.72	4.03	4.40	4.43	5.16

and believes that it can be approximated by a power law of the form

$$y_i \simeq K(x_i)^{\alpha}\,, \qquad \text{for} \quad i = 1, 2, \ldots, 8\,, \tag{3}$$

for some constants K and α. By taking the (natural) logarithm of each side of (3), show that the values of K and α can be obtained by using the method of least squares to fit a straight line whose slope and intercept you must specify. Determine the approximate numerical values of K and α that follow from pursuing this approach (express your final answer to **3 significant figures**).

[15 marks]

5. Calculate the Taylor expansion of the function

$$f(x, y) = \left(\frac{x}{y}\right)^{x+1}\,, \qquad (x > 0,\, y > 0)\,,$$

about the point $(1, 1)$ up to and including quadratic terms.

[15 marks]

COURSEWORK MARKING CRITERIA

The assignment will be marked out of 100. The questions do not carry equal weight.

You should read the question carefully and ensure that you answer each element of the question.

You do not need to word-process your submitted answers, but you should take care over the neatness and legibility of your solutions. Marks will be awarded for legible and well-presented work.

Examine the way that model solutions are presented in textbooks and the solutions to problem sheets posted on Moodle. Use these as a guide to presenting a finished piece of work. These invariably contain explanations for trickier steps in the calculations, i.e. the maths is often interspersed with written text. You will gain credit for following this practice.

Remember that getting the $right$ answer is not enough – the process by which it is obtained is more important and demonstrates understanding. So simply writing down the answers will not gain credit. Likewise, the answer can usually be found in more than one way, so it is important that you $show\ your\ workings$. If your argument is flawed, then you will lose credit even if you obtain the correct final answer.