

Class Test Revision

30 March 2019

1. Find the integrating factors for the following linear ODEs:

$$x(x+1)\frac{dy}{dx} - y = 2x^2(x+1);$$

$$\frac{dy}{dx} - 2xy = x; \quad \frac{dy}{dx} + x^2y = x^3.$$

2. Find the solution of the initial-value problem

$$x\frac{dy}{dx} = x^2 + 3y, \quad y(1) = 2.$$

3. Consider the function

$$f(x, y, z) = \frac{x + z}{y + z}.$$

Calculate $f_{zy}(3, 1, 2)$.

4. Find the value of $\alpha \in \mathbb{R}$ for which the vectors

$$\mathbf{a} = (1, -1, 3), \quad \mathbf{b} = (2, 0, 2\alpha + 1), \quad \mathbf{c} = (8, -2, 21).$$

are coplanar

5. Find the Taylor expansion of the function $f(x, y) = \sqrt{y+1} \sin(x^2)$ about the point $(\sqrt{\pi/2}, 1)$, up to and including quadratic terms.

6. Find the least squares line of best fit for the data points $(0, 4)$; $(-1, 0)$; $(1, 6)$.

7. A cylindrical container with a constant cross-section of A square feet is filled with water to a height of h ft. The water is then allowed to flow out through an orifice of cross-section a square inches located at the base of the container. The time (in seconds) that it takes to empty the tank is given by

$$T = \frac{A}{a} \sqrt{\frac{2h}{g}},$$

where g is the constant of acceleration.

Assuming that A increases by 0.1%, a decreases by 0.2% and h increases by 0.4%, find the approximate relative change in T .

8. Find the normal to the plane that contains the lines

$$\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-2}{-1} \quad \text{and} \quad \frac{x+2}{4} = \frac{y-5}{6} = \frac{z-2}{-2}.$$

9. Consider the function $f(x, y) = x^2 - y^2$, where

$$x(v, w) = e^v \sin w, \quad y(v, w) = e^w \cos v.$$

Then

1. $\frac{\partial f}{\partial v} = e^{2v} \sin^2 w + e^{2w} \sin v \cos v$;
2. $\frac{\partial f}{\partial v} = 2e^{2v} \sin^2 w + e^{2w} \sin 2v$;
3. $\frac{\partial f}{\partial v} = e^{2v} \sin w \cos w - e^{2w} \cos^2 v$;
4. $\frac{\partial f}{\partial v} = e^{2v} \sin 2w - 2e^{2w} \cos^2 v$;
5. None of the above.