The University of Nottingham

SCHOOL OF MATHEMATICAL SCIENCES

A LEVEL 1 MODULE, SPRING SEMESTER 2018-2019

MATHEMATICAL METHODS

CLASS TEST

Time allowed FORTY Minutes

Candidates must NOT start writing until the start of the test period is announced.

Answer all TEN multiple choice questions. Responses must be made on the response sheet provided

Only silent, self-contained calculators with a Single-Line Display or Dual-Line Display are permitted in this examination.

Dictionaries are not allowed with one exception. Those whose first language is not English may use a standard translation dictionary to translate between that language and English provided that neither language is the subject of this examination. Subject specific translation dictionaries are not permitted.

No electronic devices capable of storing and retrieving text, including electronic dictionaries, may be used.

DO NOT turn test paper over until instructed to do so

1. The angle between the planes

$$5(x + z) = 7 - 2y \quad \text{and} \quad 2x + 8y + 2z = 17$$

is

(a) $\sin^{-1} \dfrac{1}{\sqrt{3}}$;

(b) $\cos^{-1} \dfrac{2}{\sqrt{6}}$;

(c) $\cos^{-1} \dfrac{1}{\sqrt{3}}$;

(d) $\sin^{-1} \dfrac{2}{\sqrt{6}}$;

(e) none of the above.   [4 marks]

2. The vectors $u = -i + (\alpha - 3)j + 3k$, $v = \alpha i + j - 4k$, $w = 2i + 7j + 5k$ are coplanar for

(a) $\alpha = 5$ and $\alpha = \dfrac{3}{7}$;

(b) $\alpha = -5$;

(c) $\alpha = -5$ and $\alpha = \dfrac{2}{5}$;

(d) $\alpha = 5$ and $\alpha = \dfrac{3}{5}$;

(e) none of the above.   [4 marks]

3. A normal vector to the plane containing the parallel lines

$$\frac{x-1}{3} = -y = \frac{1-z}{2} \qquad \text{and} \qquad \frac{x}{6} = \frac{y-2}{-2} = \frac{-z}{4}$$

is

(a) $(1, -1, 1)$;

(b) $(1, 1, 1)$;

(c) $(1, -1, -1)$;

(d) $(-5, -5, 5)$;

(e) none of the above. [4 marks]

4. Consider the function

$$f(x, y) = \frac{xy}{x + \sqrt{y}} \, .$$

Then, $f_{xy}(2, 16)$ is equal to

(a) $\dfrac{1}{4}$;

(b) $\dfrac{5}{54}$;

(c) $\dfrac{3}{48}$;

(d) $\dfrac{1}{6}$;

(e) none of the above. [4 marks]

5. Consider the partial differential equation

$$2x\frac{\partial f}{\partial x} - y\frac{\partial f}{\partial y} = (xy)^3 \sin\left(\frac{x}{y}\right).$$

If the equation is re-written in terms of the variables (u, v), with

$$u = x^2 y, \quad v = xy^2,$$

then the transformed equation is

(a) $\dfrac{\partial f}{\partial u} - \dfrac{\partial f}{\partial v} = v \sin\left(\dfrac{u}{v}\right);$

(b) $(v - u)\dfrac{\partial f}{\partial u} = \sin\left(\dfrac{u}{v}\right);$

(c) $\dfrac{\partial f}{\partial u} = \dfrac{v}{3} \sin\left(\dfrac{u}{v}\right);$

(d) $\dfrac{\partial f}{\partial v} = \dfrac{u}{3} \sin\left(\dfrac{u}{v}\right);$

(e) none of the above. [4 marks]

6. The specific gravity (S) of an object with density greater than that of water can be determined by using the formula

$$S = \frac{A}{A - W},$$

where A and W are the weights of the object in air and water, respectively. If the measurements of an object are $A = 2.2$ Kg and $W = 1.8$ Kg with maximum relative errors of 0.3% and 0.1%, respectively, the maximum relative error in calculating S is approximately

(a) 0.4%;

(b) 1.8%;

(c) 4%;

(d) 1.2%;

(e) none of the above. [4 marks]

7. The Taylor expansion of the function $f(x, y) = 3e^{x^2-y} \cos\left(\dfrac{x - y^2}{3}\right)$ about the point $(1, 1)$, up to and including linear terms, is

(a) $3 + 6x - 3y + \ldots$;

(b) $6x + 3y + \ldots$;

(c) $3 - 6x + 3y + \ldots$;

(d) $6x - 3y + \ldots$;

(e) none of the above. [4 marks]

8. The slope of the least squares best-fit line for the data points $(0, 3)$, $(-2, 0)$, $(3, 6)$ is

(a) $\dfrac{37}{38}$;

(b) $\dfrac{43}{27}$;

(c) $\dfrac{13}{38}$;

(d) $\dfrac{45}{48}$;

(e) none of the above.   [4 marks]

9. The solution of the initial-value problem

$$x\frac{\mathrm{d}y}{\mathrm{d}x} + (x+1)y = x, \qquad y(\ln 2) = 1, \quad (x > 0)$$

is

(a) $e^x - 4e^{-x} + 1$;

(b) $x + 1 - 2e^{-x}$;

(c) $\dfrac{x - 1 + 2e^{-x}}{x}$;

(d) $\dfrac{\cosh x - x \sinh x}{e^x}$;

(e) none of the above. [4 marks]

10. The solution of the differential equation

$$(1 - y^2)\sin x \frac{dy}{dx} - y\cos x = 0,$$

which satisfies $y = 1$ when $x = \pi/2$, is given by

(a) $\ln|y| + \dfrac{1}{2}(1 - y^2) = \ln|\sin x|$;

(b) $e^y + \dfrac{1}{2}y^2 = \ln|\cos x|$;

(c) $\ln|y| - \dfrac{1}{2}(1 - y^2) = \ln|\cos x|$;

(d) $\ln|x| + \dfrac{1}{2}y^2 = \ln|\sin x|$;

(e) none of the above. [4 marks]