The University of Nottingham

SCHOOL OF MATHEMATICAL SCIENCES

A LEVEL 1 MODULE, SPRING SEMESTER 2017-2018

ENGINEERING MATHEMATICS 2

CLASS TEST

Time allowed FORTY Minutes

Candidates must NOT start writing until the start of the test period is announced.

Answer all TEN multiple choice questions. Responses must be made on the response sheet provided

Only silent, self-contained calculators with a Single-Line Display or Dual-Line Display are permitted in this examination.

Dictionaries are not allowed with one exception. Those whose first language is not English may use a standard translation dictionary to translate between that language and English provided that neither language is the subject of this examination. Subject specific translation dictionaries are not permitted.

No electronic devices capable of storing and retrieving text, including electronic dictionaries, may be used.

DO NOT turn test paper over until instructed to do so

1. The two lines

$$(\mathcal{L}_1): \quad \frac{x-1}{3} = y-2 = \frac{z+3}{-2},$$

$$(\mathcal{L}_2): \quad \frac{x}{5} = -\frac{y+7}{9} = \frac{z-5}{3}$$

are

(a) parallel;
(b) right-handed;
(c) orthogonal;
(d) intersecting;
(e) none of the above.   [4 marks]

2. The points A, B and C are the vertices of a triangle. If $\overrightarrow{AB} = (2, -1, 3)$ and $\overrightarrow{AC} = (1, 2, -2)$, then the area of the triangle ABC is equal to

(a) $\sqrt{80}$;

(b) $\dfrac{\sqrt{10}}{3}$;

(c) $3\sqrt{10}$;

(d) $3\sqrt{\dfrac{5}{2}}$;

(e) none of the above. [4 marks]

3. If $a = i + j + 3k$, $b = -i + 2j + k$ and $c = 4i - j - k$, then the value of $a \cdot (b \times c)$ is

(a) 0;

(b) 10;

(c) 19;

(d) -21;

(e) none of the above. [4 marks]

4. If $u(x, y) = x^5 + 6xy + y^2$ and $v(x, y) = x^2 - 5x^4y - 3y^2$, then

    (a) $\dfrac{\partial v}{\partial x} + \dfrac{\partial u}{\partial y} = 0$;

    (b) $\dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} = 0$;

    (c) $\dfrac{\partial^2 u}{\partial x^2} = \dfrac{\partial^2 v}{\partial y^2}$;

    (d) $\dfrac{\partial^2 u}{\partial x^2} = 0$;

    (e) none of the above.   [4 marks]

5. Consider $a \in \mathbb{R}$ (an arbitrary real number) and the function

$$f(x, y) = \frac{e^{x/y} - e^{y/x}}{e^x + e^{-y}} .$$

Then, $f_x(a, a)$ is equal to

(a) 0;

(b) $\dfrac{2e^a}{e^{2a} + 1}$;

(c) $\dfrac{2e^{a+1}}{a(e^{2a} + 1)}$;

(d) $\dfrac{2e^{a+1}}{(e^a - e^{-a})^2}$;

(e) none of the above.   [4 marks]

6. Consider the function

$$f(u,v,w) = u^3 w\, \mathrm{e}^v + v^3 u\, \mathrm{e}^w,$$

where

$$u = x, \quad v = xy, \quad w = y^2.$$

Then

(a) $\dfrac{\partial f}{\partial x} = x^2 y^2 \left[4\mathrm{e}^{xy} + xy\left(\mathrm{e}^{xy} + 3\mathrm{e}^{y^2} \right) \right];$

(b) $\dfrac{\partial f}{\partial x} = x^2 y^2 \left[3\mathrm{e}^{xy} + xy\left(\mathrm{e}^{xy} + 4\mathrm{e}^{y^2} \right) \right];$

(c) $\dfrac{\partial f}{\partial x} = x^2 y^2 \left[3 + x^2 y^2 \left(\mathrm{e}^{xy} + 4\mathrm{e}^{y^2} \right) \right];$

(d) $\dfrac{\partial f}{\partial x} = x^3 y^2 \mathrm{e}^{xy} + x^4 y^3 \mathrm{e}^{y^2};$

(e) none of the above. [4 marks]

7. A storage tank has the shape of a right circular cone of radius R and height H. Assuming that its radius increases by 0.5% and the height decreases by 0.25%, the percentage relative change in the volume of the storage tank is

   (a) 1.25%;

   (b) 3%;

   (c) 0.75%;

   (d) 1.5%;

   (e) none of the above.   [4 marks]

8. A dragonfly follows a trajectory described by the position vector

$$\mathbf{x}(t) = \left(\ln(1 + t\sqrt{t}), \ \frac{25t + 50}{3t + 3}, \ -\frac{8}{3\pi}\cos\left(\frac{\pi t}{8}\right) \right).$$

Its speed at time $t = 4$ is

(a) $\sqrt{3}$;

(b) $\dfrac{1}{\sqrt{3}}$;

(c) $\dfrac{1}{3}$;

(d) $3\sqrt{3}$;

(e) none of the above.   [4 marks]

9. The differential equation

$$(x + y)x^2 \frac{dy}{dx} = -xy^2 - 3x^2 y$$

is

   (a) linear;
   (b) separable;
   (c) exact;
   (d) implicit;
   (e) none of the above.   [4 marks]

$$(x + y)x^2 \frac{dy}{dx} = -xy^2 - 3x^2 y$$

10. If the function $y = y(x)$ satisfies the ordinary differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y^2 - y}{x^2 + x}$$

and $C \in \mathbb{R}$ is an arbitrary constant, then

(a) $\left(1 - \dfrac{1}{y}\right)\left(1 + \dfrac{1}{x}\right) = C$;

(b) $\left(1 - \dfrac{1}{y}\right)\dfrac{x - 1}{x} = C$;

(c) $\dfrac{1}{y - 1} = \dfrac{1}{C(x + 1)}$;

(d) $y - 1 = \dfrac{Cx}{x + 1}$;

(e) none of the above.   [4 marks]