

Engineering Maths 2  
Class Test Solutions (2018-19)

Q1.  $5x + 2y + 5z = 7 \Rightarrow \underline{n}_1 = (5, 2, 5)$   
 $2x + 8y + 2z = 17 \Rightarrow \underline{n}_2 = (1, 4, 1)$

$\theta =$  angle between  
 $\underline{n}_1$  and  $\underline{n}_2$

$$\cos \theta = \frac{\underline{n}_1 \cdot \underline{n}_2}{|\underline{n}_1| |\underline{n}_2|} = \frac{18}{\sqrt{54} \sqrt{18}} = \frac{1}{\sqrt{3}}$$

$$|\underline{n}_1| = \sqrt{54}$$

$$|\underline{n}_2| = \sqrt{18}$$

$$\theta = \cos^{-1} \left( \frac{1}{\sqrt{3}} \right)$$

Hence, correct answer is (C)

Q2.  $\underline{u}, \underline{v}, \underline{w}$  coplanar  $\Leftrightarrow [\underline{u}, \underline{v}, \underline{w}] = 0$

$$\begin{vmatrix} -1 & a-3 & 3 \\ a & 1 & -4 \\ 2 & 7 & 5 \end{vmatrix} = 0 \Leftrightarrow \dots \Leftrightarrow 5a^2 - 28a + 15 = 0$$

$$a_{1,2} = 5, \frac{3}{5}$$

Hence, correct answer is (D)

Q3.  $\frac{x-1}{3} = \frac{y}{-1} = \frac{z-1}{-2}$   $\begin{cases} \rightarrow A(1, 0, 1) \text{ on the line} \\ \rightarrow \underline{d}_1 = (3, -1, -2) \text{ direction vector} \end{cases}$

$\frac{x}{6} = \frac{y-2}{-2} = \frac{z}{-4}$   $\begin{cases} \rightarrow B(0, 2, 0) \text{ on the line} \\ \rightarrow \underline{d}_2 = (6, -2, -4) = 2(3, -1, -2) \text{ direction vector} \end{cases}$

$$\vec{BA} = (1, 0, 1) - (0, 2, 0) = (1, -2, 1)$$

Normal vector =  $\vec{BA} \times \underline{d}_1 = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{vmatrix} = \dots = 5(1, 1, 1)$

Hence, correct answer is (B)

$$Q4. f(x,y) = \frac{xy}{x+\sqrt{y}}$$

$$f_x = \frac{1}{(x+\sqrt{y})^2} [y(x+\sqrt{y}) - xy(1)] = \frac{y\sqrt{y}}{(x+\sqrt{y})^2} = \frac{y^{3/2}}{(x+\sqrt{y})^2}$$

$$f_{xy} = \frac{1}{(x+\sqrt{y})^4} \left[ \frac{3}{2} y^{1/2} (x+\sqrt{y})^2 - y^{3/2} \cdot 2(x+\sqrt{y}) \cdot \frac{1}{2\sqrt{y}} \right]$$

$$= \dots = \frac{3x\sqrt{y} + y}{2(x+\sqrt{y})^3}$$

$$f_{xy}(2,16) = \frac{3(2)(4) + 16}{2(2+4)^3} = \frac{40}{2 \cdot 6^3} = \frac{5}{54}$$

Hence, correct answer is (B)

$$Q5. 2x \frac{\partial f}{\partial x} - y \frac{\partial f}{\partial y} = x^3 y^3 \sin\left(\frac{x}{y}\right) \quad (*)$$

$$u = x^2 y$$

$$v = xy^2$$

$$\frac{\partial u}{\partial x} = 2xy$$

$$\frac{\partial u}{\partial y} = x^2$$

$$\frac{\partial v}{\partial x} = y^2$$

$$\frac{\partial v}{\partial y} = 2xy$$

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial u} \frac{\partial u}{\partial x} + \frac{\partial f}{\partial v} \frac{\partial v}{\partial x} = (2xy) \frac{\partial f}{\partial u} + (y^2) \frac{\partial f}{\partial v}$$

$$\frac{\partial f}{\partial y} = \frac{\partial f}{\partial u} \frac{\partial u}{\partial y} + \frac{\partial f}{\partial v} \frac{\partial v}{\partial y} = (x^2) \frac{\partial f}{\partial u} + (2xy) \frac{\partial f}{\partial v}$$

Substituting into (\*):

$$\begin{aligned} 2x \frac{\partial f}{\partial x} - y \frac{\partial f}{\partial y} &= 4x^2 y \frac{\partial f}{\partial u} + 2xy^2 \frac{\partial f}{\partial v} - x^2 y \frac{\partial f}{\partial u} - 2xy^2 \frac{\partial f}{\partial v} \\ &= 3x^2 y \frac{\partial f}{\partial u} = 3u \frac{\partial f}{\partial u} \end{aligned}$$

$$\text{So } 3u \frac{\partial f}{\partial u} = uv \sin\left(\frac{u}{v}\right) \Rightarrow \frac{\partial f}{\partial u} = \frac{v}{3} \sin\left(\frac{u}{v}\right) \Rightarrow \text{Correct answer (C)}$$

$$Q6. S = f(A, W)$$

$$f(A, W) \equiv \frac{A}{A - W}$$

$$\varepsilon_S^{\max} = \left| \frac{A_0 f_A^0}{f_0} \right| |\varepsilon_A| + \left| \frac{W_0 f_W^0}{f_0} \right| |\varepsilon_W|$$

$$f_A = -\frac{W}{(A - W)^2}$$

$$f_W = \frac{A}{(A - W)^2}$$

$$\frac{A_0 f_A^0}{f_0} = A_0 \cdot \left[ -\frac{W_0}{(A_0 - W_0)^2} \right] \cdot \frac{A_0 - W_0}{A_0} = -\frac{W_0}{A_0 - W_0}$$

$$\frac{W_0 f_W^0}{f_0} = W_0 \cdot \left[ \frac{A_0}{(A_0 - W_0)^2} \right] \cdot \frac{A_0 - W_0}{A_0} = \frac{W_0}{A_0 - W_0}$$

$$\varepsilon_S^{\max} = \left| \frac{W_0}{A_0 - W_0} \right| (|\varepsilon_A| + |\varepsilon_W|) = \frac{1.8}{0.4} (0.3\% + 0.1\%) = 1.8\%$$

Hence, correct answer is (B)

$$Q7. f(x, y) = 3e^{x^2 - y} \cos\left(\frac{x - y^2}{3}\right) \quad (x_0, y_0) = (1, 1)$$

$$f_x = 3(2x)e^{x^2 - y} \cos\left(\frac{x - y^2}{3}\right) + 3e^{x^2 - y} \left(-\sin\left(\frac{x - y^2}{3}\right)\right) \left(\frac{1}{3}\right)$$

$$\Rightarrow \boxed{f_x^0 = 6}$$

$$f_y = -3e^{x^2 - y} \cos\left(\frac{x - y^2}{3}\right) + 3e^{x^2 - y} \cdot \left(-\sin\left(\frac{x - y^2}{3}\right)\right) \left(-\frac{2y}{3}\right)$$

$$\Rightarrow \boxed{f_y^0 = -3}$$

$$f(x_0, y_0) = 3$$

$$f(x, y) = f(x_0, y_0) + f_x^0(x - 1) + f_y^0(y - 1) + \dots$$

$$= 3 + 6(x - 1) - 3(y - 1) + \dots = 6x - 3y + \dots$$

Hence, correct answer is (D)

Q8. 

$x_i$	$y_i$
0	3
-2	0
3	6

$$S(a,b) = \sum_{i=1}^3 (y_i - ax_i - b)^2$$

$$\Rightarrow S(a,b) = (3-b)^2 + (2a-b)^2 + (6-3a-b)^2$$

$$\frac{\partial S}{\partial a} = 0 \Leftrightarrow \dots \Leftrightarrow 13a + b = 18$$

$$\frac{\partial S}{\partial b} = 0 \Leftrightarrow \dots \Leftrightarrow a + 3b = 9$$

$\rightarrow$  (Solve for a & b)  $\rightarrow$   $\boxed{a = \frac{45}{38}}$

Hence, correct answer is (E)

Q9.  $\frac{dy}{dx} + \left(\frac{x+1}{x}\right)y = 1$        $y(\ln 2) = 1$       ( $x > 0$ )

$$R(x) = \exp\left(\int \frac{x+1}{x} dx\right) = \exp\left(\int dx + \int \frac{dx}{x}\right) = e^{x+\ln x} = xe^x$$

$$y(x) = \frac{1}{R(x)} \int R(x) \cdot 1 dx + \frac{C}{R(x)} = \frac{1}{x} e^{-x} (xe^x - e^x) + \frac{C}{x} e^{-x}$$

$$\int R(x) dx = \int xe^x dx = \boxed{xe^x - e^x} = \frac{1}{x}(x-1) + \frac{C}{x} e^{-x}$$

$$= \frac{1}{x}(x-1 + Ce^{-x})$$

$$y(\ln 2) = \frac{1}{\ln 2} (\ln 2 - 1 + \frac{C}{2}) = 1 \Rightarrow \ln 2 - 1 + \frac{C}{2} = \ln 2 \Rightarrow \boxed{C = 2}$$

So  $y = \frac{1}{x}(x-1+2e^{-x})$

Hence, correct answer is (C)

Q10.  $\frac{(1-y^2)}{y} dy = \frac{\cos x}{\sin x} dx \Rightarrow \int \left(\frac{1}{y} - y\right) dy = \int \frac{\cos x}{\sin x} dx$

$$\Rightarrow \ln|y| - \frac{1}{2}y^2 = + \ln|\sin x| + C \quad (C = \text{integration const.})$$

$$y=1 \text{ when } x = \frac{\pi}{2} \Rightarrow C = -\frac{1}{2} \Rightarrow \text{correct answer is } \boxed{\text{(A)}}$$