Engineering Maths 2
Class Test Solutions (2017/18)

Q.1. Direction vectors: $\underline{d}_1 = (3, 1, -2)$ $\underline{d}_2 = (5, -9, 3)$

$\underline{d}_1 \cdot \underline{d}_2 = (3)(5) + (1)(-9) + (-2)(3) = 15 - 9 - 6 = 0$

Hence the lines are orthogonal. Correct answer is Ⓒ

Q.2. Area $\triangle ABC = \frac{1}{2} |\overrightarrow{AB} \times \overrightarrow{AC}|$

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{K} \\ 2 & -1 & 3 \\ 1 & 2 & -2 \end{vmatrix} = \underline{i}(2-6) + \underline{j}(3+4) + \underline{K}(4+1)$$
$$= -4\underline{i} + 7\underline{j} + 5\underline{K}$$

$|\overrightarrow{AB} \times \overrightarrow{AC}| = \sqrt{(-4)^2 + 7^2 + 5^2} = \sqrt{16 + 49 + 25} = \sqrt{90} = 3\sqrt{10}$

Hence the required area is $\frac{3\sqrt{10}}{2} = 3\sqrt{\frac{5}{2}}$. Correct answer is Ⓓ

Q.3.

$$\underline{a} \cdot (\underline{b} \times \underline{c}) = \begin{vmatrix} 1 & 1 & 3 \\ -1 & 2 & 1 \\ 4 & -1 & -1 \end{vmatrix} = (1)\begin{vmatrix} 2 & 1 \\ -1 & -1 \end{vmatrix} - (1)\begin{vmatrix} -1 & 1 \\ 4 & -1 \end{vmatrix} + (3)\begin{vmatrix} -1 & 2 \\ 4 & -1 \end{vmatrix}$$
$$= (-2+1) - (1-4) + 3(1-8)$$
$$= -19$$

Correct answer is Ⓔ

Q.4. $u_x = 5x^4 + 6y$ $\qquad u_y = 6x + 2y$

$v_x = 2x - 10x^3 y$ $\qquad v_y = -5x^4 - 6y$

Clearly $u_x + v_y = 0$

Correct answer is Ⓑ

Q.5. $f_x = \dfrac{1}{(e^x + e^{-y})^2}\left[\dfrac{\partial}{\partial x}\left(e^{\frac{x}{y}} - e^{\frac{y}{x}}\right)\cdot\left(e^x + e^{-y}\right) - \left(e^{\frac{x}{y}} - e^{\frac{y}{x}}\right)\cdot\dfrac{\partial}{\partial x}\left(e^x + e^{-y}\right)\right]$

$= \dfrac{1}{(e^x + e^{-y})^2}\left[\left(\dfrac{1}{y}e^{\frac{x}{y}} + \dfrac{y}{x^2}\cdot e^{\frac{y}{x}}\right)\left(e^x + e^{-y}\right) - \left(e^{\frac{x}{y}} - e^{\frac{y}{x}}\right)\cdot e^x\right]$

$f_x(a,a) = \dfrac{1}{(e^a + e^{-a})^2}\cdot\left[\left(\dfrac{1}{a}e + \dfrac{1}{a}e\right)\left(e^a + e^{-a}\right) - 0\right]$

$= \dfrac{2}{a}\cdot\dfrac{e}{e^a + e^{-a}} = \dfrac{2e^{a+1}}{a\left(e^{2a}+1\right)}$

Correct answer is Ⓒ

Q.6. $\dfrac{\partial f}{\partial x} = \dfrac{\partial f}{\partial u}\dfrac{\partial u}{\partial x} + \dfrac{\partial f}{\partial v}\dfrac{\partial v}{\partial x} + \cancel{\dfrac{\partial f}{\partial w}\dfrac{\partial w}{\partial x}}$ (w does NOT depend on x)

$\dfrac{\partial f}{\partial u} = 3u^2 w\, e^v + v^3 e^w$ $\dfrac{\partial f}{\partial v} = u^3 w\, e^v + 3v^2 u\, e^w$

$\dfrac{\partial u}{\partial x} = 1$ $\dfrac{\partial v}{\partial x} = y$

So $\dfrac{\partial f}{\partial x} = \left(3x^2 y^2 e^{xy} + x^3 y^3 e^{y^2}\right)\cdot(1) + \left(x^3 y^2 e^{xy} + 3x^2 y^2 (x)\, e^{y^2}\right)(y)$

$= 3x^2 y^2 e^{xy} + \underline{x^3 y^3 e^{y^2}} + x^3 y^3 e^{xy} + \underline{3x^3 y^3 e^{y^2}}$

$= 3x^2 y^2 e^{xy} + 4x^3 y^3 e^{y^2} + x^3 y^3 e^{xy}$

$= x^2 y^2\left[3\, e^{xy} + xy\left(e^{xy} + 4 e^{y^2}\right)\right]$

Correct answer is Ⓑ

Q.7. $V = \frac{1}{3}\pi R^2 H = f(R, H)$

$f_R = \frac{2}{3}\pi R H$ $\qquad f_H = \frac{1}{3}\pi R^2$

$\varepsilon_R = +0.5\%$ $\qquad \varepsilon_H = -0.25\%$

$\varepsilon_f = \left(\frac{R_0 f_R^0}{f_0}\right)\varepsilon_R + \left(\frac{H_0 f_H^0}{f_0}\right)\varepsilon_H = 2 \times (0.5\%) + 1 \times (-0.25\%) = 0.75\%$

$\frac{R_0 f_R^0}{f_0} = R_0 \left(\frac{2}{3}\pi R_0 H_0\right) \cdot \frac{3}{\pi R_0^2 H_0} = 2$

$\frac{H_0 f_H^0}{f_0} = H_0 \left(\frac{1}{3}\pi R_0^2\right) \cdot \frac{3}{\pi R_0^2 H_0} = 1$

correct
answer is Ⓒ

Q.8. $\underline{V}(t) = \frac{d\underline{X}}{dt} = \left(\frac{1}{1+t\sqrt{t}}\cdot\frac{3}{2}t^{1/2}, \; \frac{25}{3}\cdot\left(-\frac{1}{(t+1)^2}\right), \; -\frac{8}{3\pi}\cdot\frac{\pi}{8}\cdot-\sin\left(\frac{\pi t}{8}\right)\right)$

$\underline{V}(t=4) = \left(\frac{1}{\cancel{9}_3}\cdot\frac{\cancel{3}}{2}\cdot\cancel{2}, \; \frac{25}{3}\cdot\left(-\frac{1}{\cancel{25}}\right), \; +\frac{1}{3}\sin\frac{\pi}{2}\right)$

$= \left(\frac{1}{3}, \; -\frac{1}{3}, \; +\frac{1}{3}\right)$

speed $= |\underline{V}| \Rightarrow$ required speed $= \sqrt{\frac{1}{3^2}+\frac{1}{3^2}+\frac{1}{3^2}} = \frac{1}{\sqrt{3}} \Rightarrow$ correct answer is Ⓑ

Q.9. $\underbrace{(xy^2 + 3x^2 y)}_{M(x,y)} + \underbrace{(x+y)x^2}_{N(x,y)}\frac{dy}{dx} = 0$

$\frac{\partial M}{\partial y} = 2xy + 3x^2$ $\qquad\qquad \frac{\partial N}{\partial x} = 3x^2 + 2xy \Rightarrow \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

i.e. ODE is exact

Correct answer is Ⓒ

Q.10. $\dfrac{dy}{y^2-y} = \dfrac{dx}{x^2+x}$

$$\dfrac{1}{y^2-y} = \dfrac{1}{y-1} - \dfrac{1}{y}$$

$$\dfrac{1}{x^2+x} = \dfrac{1}{x} - \dfrac{1}{x+1}$$

$\Rightarrow \displaystyle\int \left(\dfrac{1}{y-1} - \dfrac{1}{y}\right) dy = \int \left(\dfrac{1}{x} - \dfrac{1}{x+1}\right) dx$

$\Rightarrow \ln \dfrac{y-1}{y} = \ln \dfrac{x}{x+1} + \ln C \Rightarrow \ln \dfrac{y-1}{y} = \ln \dfrac{Cx}{x+1}$

$\Rightarrow \dfrac{y-1}{y} = \dfrac{Cx}{x+1} \Rightarrow 1 - \dfrac{1}{y} = \dfrac{C}{\frac{x+1}{x}} = \dfrac{C}{1+\frac{1}{x}} \Rightarrow \left(1-\dfrac{1}{y}\right)\left(1+\dfrac{1}{x}\right) = C$

Correct answer is (A)