SOME IMPORTANT LIMITS

1.

$$\lim_{x\to\infty}\frac{\mathrm{e}^x}{x^n}=+\infty \qquad \text{and} \qquad \lim_{x\to\infty}x^n\mathrm{e}^{-x}=0 \qquad (n=0,1,2,\dots)$$

2.

$$\lim_{x\to\infty}\ln x=+\infty \qquad \text{and} \qquad \lim_{x\to 0^+}\ln x=-\infty$$

3.

$$\lim_{x\to\infty}\frac{\ln x}{x^\alpha}=0\,, \qquad \text{for all } \alpha>0$$

4.

$$\lim_{x\to 0}\frac{\mathrm{e}^x-1}{x}=1$$

5.

$$\lim_{x\to 0}\frac{\sin x}{x}=1$$

October 2017
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LIMITS AT $\pm\infty$ FOR RATIONAL FUNCTIONS

$$P(x) = a_m x^m + a_{m-1}x^{m-1} + \cdots + a_1 x + a_0\,, \qquad (a_0, a_1, \ldots, a_m \in \mathbb{R})$$

$$Q(x) = b_n x^n + b_{n-1}x^{n-1} + \cdots + b_1 x + b_0\,, \qquad (b_0, b_1, \ldots, b_n \in \mathbb{R})$$

Note that $\deg(P) = m$ and $\deg(Q) = n$.

If $m < n$ then

$$\lim_{x\to+\infty} \frac{P(x)}{Q(x)} = 0\,.$$

If $m = n$ then

$$\lim_{x\to+\infty} \frac{P(x)}{Q(x)} = \frac{a_m}{b_m}\,.$$

If $m > n$ then

$$\lim_{x\to+\infty} \frac{P(x)}{Q(x)} = \operatorname{sgn}\left(\frac{a_m}{b_n}\right) \times (+\infty)\,.$$

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