Feedback for Coursework Assignment 2

The coursework was marked out of 100. The standard of the answers was generally very good, so well done to those who invested time and effort in what they submitted.

- **Question 1**

  1. This was answered very well by most students. The most common mistake was forgetting to add the ellipsis points after the last non-zero calculated term and mentioning that the expansion is valid "as $x \to 0$" or "for small x". One mark was lost for these omissions.

  2. While many got the correct final answer, my impression was that some over-complicated the solution. No penalties were applied this time, but this might change when it comes to marking the exam paper. Typically, some students calculated from scratch the Maclaurin expansion of the functions $\ln(1+x^3)$, e^{2x}, etc. Please note that this was not necessary; if you have the Maclaurin expansion for $\ln(1+x)$, then just make the substitution $x \to x^3$ in that result. The same applies to e^{2x}: in the expansion of e^x (on the Formula Sheet), you let $x \to 2x$, and that will give you the desired result.
  There were some minor problems with the notation as well. For example, some wrote things like
  $$\sin x \simeq x - \frac{x^3}{3!}\,.$$
  This is meaningless, unless you specify that the approximation holds for $x \to 0$ (i.e., "for small x"). I suggest using the model solutions posted on Moodle (I made exactly the same comment last year – you can check the feedback for last year's CW2).
  Some students multiplied out the expansions in the numerator of the first fraction and they also squared out the corresponding expressions in the denominator. It is not wrong (and I admire your tenacity), but please bear in mind that it serves no purpose and it increases the risk of making a mistake along the way. Just pull out the appropriate common factors, simplify the fraction, and then take the limit.

  Action: Review the material on Maclaurin expansions and the various limits worked out in class. Study the past CW and the recommended textbooks to see how the solutions should be written.

- **Question 2**: Many good solutions for all three integrals.

  1. The most common error for this one was to leave the final result in terms of the u-substitution. Please remember, after the u-integral is evaluated you need to transform the result in terms of x (because that's the independent variable in the original integral). Minor penalties were applied for not finishing the integral properly.

2. Apparently, this was the most challenging of the three integrals. Some made the substitution $u = x^4/4$. While there is nothing wrong with this, it does make the denominator of the u-integral difficult to "read", and the subsequent partial fraction decomposition of the corresponding integral presents more opportunities for introducing small errors. Some students made the correct substitution, but failed to use the partial decomposition to evaluate the integral. In some cases everything was executed correctly, except for a wrong multiplicative constant in the final result (see Moodle for the correct answer).

3. Some students used the substitutions $u = 5 - \cos^2 x$ or $u = \cos^2 x$. These lead to fairly awkward u-integrals that require further substitutions before the correct answer starts to emerge. No penalties were applied for doing this, but keep in mind that in the exam you will not have time to go in circles and guess what the right substitution should be. The purpose of a substitution is to reduce the integral to something simple that you can get from the Formula Sheet or by using a different integration technique. In this module you will not be asked to use repeated substitutions – if you need to make a substitution the reduced integral will be either obvious or it will require a different integration technique (parts or partial fractions).

Action: Please study the solution posted on Moodle and practice additional integrals from the recommended textbooks. Seek help if you struggle with this important topic.

- **Question 3**: The easiest question in this coursework. Sadly, many lost valuable marks by not reading the info posted on Moodle. Common errors:

  - lack of a suitable sketch to indicate what is actually being calculated;
  - identifying the wrong area (which was then calculated correctly);
  - no details regarding the intersection points between the parabola and the line;
  - some simply integrated between 0 and 6 the difference of the two functions – this is a serious error.

Each type of mistake outlined above attracted some light penalties.

Action: I deliberately spent some extra time pointing out how to tackle this type of question. Please check on Moodle the additional slides for this topic. Also, consult MathTutor for some very similar examples to the one you had in this coursework.

- **Question 4**: This one went very well, at least the first part.

1. The integration by parts formula was applied correctly by many. In a few places I have noticed that some students made little distinction between

$$\ln(x^n) \qquad \text{and} \qquad (\ln x)^n.$$

Please note that they are different, the first one is just $n \ln x$. A similar mistake was made by a different student who wrote

$$\int 5(\ln x)^4 \, dx = 20 \int \ln x \, dx,$$

which is clearly wrong. Avoid writing things like this as they will attract serious penalties in the exam.

A small minority managed to calculate without any problem the integral $I_n(x)$ for particular values of n, but struggled with the general case. Others made a small mistake by saying $dV = 1$; please note that

$$dV = dx \qquad \Rightarrow \qquad \frac{dV}{dx} = 1 \qquad \Rightarrow \qquad V = x,$$

(to get the last result I integrated and I left out the usual arbitrary constant).

2. While many used the recurrence derived in the first part of the question, the following mistakes appeared in *several* scripts:

   (a) small algebraic errors that led to the incorrect final answer (this attracted a light penalty);

   (b) some applied the recurrence correctly for $n = 5$ and then stopped;

   (c) some did everything correctly until they got to $I_1(x)$, and then stated that this integral does not exist (we did calculate $I_1(x) = \int \ln x \, dx$ in one of the lectures);

   (d) a few students calculated I_5 by integrating by parts several times; while no penalties were applied, this was very time consuming and suggests that you failed to recognise the relevance of the first part of the question (I don't suggest doing something like this in the exam);

Action: Read the questions carefully and try to understand their relevance. Last year's CW2 involved a similar question in which the students were required to find a recurrence for a different integral, and they were then asked to calculate one of the terms in the sequence for a particular value of n. Please make yourselves familiar with all the material posted on Moodle (especially the stuff from last year).

- **Question 4**: Perhaps the hardest question in CW2; surprisingly, it went very well. A small minority did not apply correctly the chain rule for T_x and T_y, and those errors propagated down in the solution contributing to the loss of valuable points. Please note that if $u = u(x, y)$ then

$$\frac{\partial}{\partial x}\left(e^{-u}\right) = \left(-e^{-u}\right)\frac{\partial u}{\partial x}.$$

Action: Read the solution posted on Moodle and compare it to your attempt. Look at last year's CW2 for a a very similar (slightly easier) example.