



The University of  
**Nottingham**

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# **HG1MC1** Mathematics for Chemistry 1

**MATH/1011/01**

(2018/19)

**Lecturer and Module Convenor:** Dr C D Coman [Ciprian.Coman@nottingham.ac.uk](mailto:Ciprian.Coman@nottingham.ac.uk)  
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**HG1MC1 Mathematics**  
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If you need to contact the lecturer with regard to the module, then email to arrange a time.

**Office Hours:**

Wednesdays 10:00-14:00 (no appointment required)

**Lecture Times:** Mondays at 16:00–17:00 (Physics C27) and Wednesdays at 9:00-10:00 (Chemistry C15). There are *two exceptions*:

In **week 2** the Wednesday morning lecture will take place on Thursday at 11:00-12:00 (Chemistry C15) – please note that this is a one-off arrangement imposed by the School of Chemistry.

In **week 3** the Wednesday lecture is cancelled.

**Workshops:** These take place on Mondays at 17:00-18:00 (Physics C27) – they will not start until about week 3-4 (you will be informed later when these are supposed to start).

These workshops are for students to practise solving problems and to receive help on the module from supporting members of staff.

**Pre-requisites:** A basic mathematical education as provided by a pass grade in A-level Mathematics or AS-level Pure Mathematics or equivalent

**Aim:** To give students of Chemistry a basic knowledge of the main mathematical techniques required in following a chemistry-based course.

**Content:**

The module provides definition, manipulation and graphical representation of important functions commonly occurring in science. The calculus of a single-variable is reviewed and extended to develop techniques of differential and integral calculus together with solution of first-order differential equations. The concept of partial differential equations is introduced.

- Functions of a single variable
- Differential calculus of a single variable
- Integral calculus of a single variable
- Differential calculus of more than one variable
- First-order differential equations
- Elementary probability and statistics

Examples in the context of chemistry are used (to the extent that the material permits).

**Assessment:** The module will be assessed by a 2-hour final **examination (80%)** and **coursework (20%)**

The examination paper will consist of FIVE questions and credit will be given for the best FOUR answers, with all five questions carrying equal marks.

The coursework will consist of TWO assignments. The DEADLINES for submission are **Monday 5th November** and **Monday 26th November** for Assessed Coursework 1 and 2, respectively. They will be set 2 weeks before the deadline.

**Web Support:** All module handouts and many other support materials are available on our web system MOODLE. This is accessed from

<http://www.moodle.nottingham.ac.uk>

by using your university username and password.

**A more detailed version of this document is also posted on MOODLE.**

*Greek Alphabet*

$\alpha$ A alpha	$\iota$ I iota	$\rho$ P rho
$\beta$ B beta	$\kappa$ K kappa	$\sigma$ $\Sigma$ sigma
$\gamma$ $\Gamma$ gamma	$\lambda$ $\Lambda$ lambda	$\tau$ T tau
$\delta$ $\Delta$ delta	$\mu$ M mu	$\upsilon$ $\Upsilon$ upsilon
$\epsilon$ E epsilon	$\nu$ N nu	$\phi$ $\Phi$ phi
$\zeta$ Z zeta	$\xi$ $\Xi$ xi	$\chi$ X chi
$\eta$ H eta	$o$ O omicron	$\psi$ $\Psi$ psi
$\theta$ $\Theta$ theta	$\pi$ $\Pi$ pi	$\omega$ $\Omega$ omega

**Some useful identities**

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$	$\sinh(A \pm B) = \sinh A \cosh B \pm \cosh A \sinh B$
$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$	$\cosh(A \pm B) = \cosh A \cosh B \pm \sinh A \sinh B$
$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$	$\tanh(A \pm B) = \frac{\tanh A \pm \tanh B}{1 \pm \tanh A \tanh B}$
$\sin A \pm \sin B = 2 \sin \frac{1}{2}(A \pm B) \cos \frac{1}{2}(A \mp B)$	$\sinh A \pm \sinh B = 2 \sinh \frac{1}{2}(A \pm B) \cosh \frac{1}{2}(A \mp B)$
$\cos A + \cos B = 2 \cos \frac{1}{2}(A + B) \cos \frac{1}{2}(A - B)$	$\cosh A + \cosh B = 2 \cosh \frac{1}{2}(A + B) \cosh \frac{1}{2}(A - B)$
$\cos A - \cos B = 2 \sin \frac{1}{2}(A + B) \sin \frac{1}{2}(B - A)$	$\cosh A - \cosh B = 2 \sinh \frac{1}{2}(A + B) \sinh \frac{1}{2}(A - B)$

**Binomial series**

$$(a + b)^n = \sum_{r=0}^n \binom{n}{r} a^r b^{n-r} \quad \text{for } n \text{ a positive integer}$$

where  $\binom{n}{r} = \frac{n!}{(n-r)!r!}$  and  $n! = n(n-1)(n-2)\dots 2.1$

**Taylor series**

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \dots + \frac{f^{(r)}(a)}{r!}(x-a)^r + \dots$$

**Standard series expansions**

$$e^x = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \dots$$

$$\sin x = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \dots$$

$$\cos x = 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \dots$$

$$\tan x = x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + \dots \quad \text{for } -\frac{1}{2}\pi < x < \frac{1}{2}\pi.$$

## Some well known integrals

If  $\int f(x)dx = F(x) + \text{constant}$  then  $F'(x) = f(x)$

$f(x)$	$F(x)$
$\sec^2 x$	$\tan x$
$\operatorname{cosec}^2 x$	$-\cot x$
$\sec x \tan x$	$\sec x$
$\operatorname{cosec} x \cot x$	$-\operatorname{cosec} x$
$\operatorname{sech}^2 x$	$\tanh x$
$\operatorname{cosech}^2 x$	$-\coth x$
$\operatorname{sech} x \tanh x$	$-\operatorname{sech} x$
$\sec x$	$\ln(\sec x + \tan x)$
$\operatorname{cosec} x$	$\ln(\operatorname{cosec} x - \cot x)$
$\operatorname{sech} x$	$\sin^{-1}(\tanh x)$
$\operatorname{cosech} x$	$\ln\left(\tanh\left(\frac{1}{2}x\right)\right)$
$\frac{1}{(a^2 + x^2)}$	$a^{-1} \tan^{-1}(x/a)$
$\frac{1}{(a^2 - x^2)}$	$\frac{1}{2a} \ln \left  \frac{a+x}{a-x} \right $
$\frac{1}{\sqrt{a^2 + x^2}}$	$\sinh^{-1}(x/a)$
$\frac{1}{\sqrt{a^2 - x^2}}$	$\sin^{-1}(x/a)$ for $ x  <  a $
$\frac{1}{\sqrt{x^2 - a^2}}$	$\cosh^{-1}(x/a)$ for $ x  >  a $

Workshop 1

Functions of one real variable

1. Determine whether the following functions are even or odd, or neither

(a)  $x^3 + 6x$ , (b)  $x^2 + \sin x$ , (c)  $e^{x^2} \cos 2x$ ,

(d)  $\int_0^x \sin^2 y \, dy$ , (e)  $\tan^{-1} x$ .

2. Write the functions  $e^{-x} \cos bx$  and  $\left(\frac{1-x}{1+x}\right)^{1/2}$  as the sums of an even and an odd function.

3. If  $f(x) = \frac{x+1}{x-1}$  and  $g(x) = \frac{2x+5}{4x-3}$ , find  $f(g(x))$  and  $g(f(x))$

4. Find the roots of the following quadratic polynomials and sketch their graphs:

(a)  $y = x^2 - 4$  (b)  $y = -x^2 - 2x + 3$ .

5. Simplify:

(a)  $e^3 e^4$ , (b)  $\frac{e^5}{e^2}$ , (c)  $\ln x^3 - \ln x^2 + \ln x$ , (d)  $\ln e^{x^3+2} - \ln e^2$ .

6. Write the following expressions as partial fractions

(a)  $\frac{5x}{(x-2)(x+3)}$  (b)  $\frac{6}{(x+1)(x^2+5)}$

(c)  $\frac{5}{(x-2)^2(x+3)}$  (d)  $\frac{2x^3+3x^2-6x-6}{x^2+x-6}$

7. Show from the definitions of  $\sinh x$  and  $\cosh x$  that

(a)  $\cosh 2x = 2 \cosh^2 x - 1$ , (b)  $\sinh(x+y) = \sinh x \cosh y + \cosh x \sinh y$ .

8. If  $y = \sinh x$  find  $x$  in terms of  $y$  and hence show that  $\sinh^{-1} x = \ln(x + \sqrt{x^2 + 1})$ .

9. Show that  $y(x) = \tanh^{-1} x = \frac{1}{2} \ln\left(\frac{1+x}{1-x}\right)$ . What are the allowable values of  $x$ ?

10. Verify that the Hermite polynomials satisfy the symmetry condition

$$H_n(-x) = (-1)^n H_n(x)$$

11. Verify the formula for Hermite polynomials  $H_n(x)$

$$\sum_{k=0}^n \binom{n}{k} H_k(x) (2y)^{n-k} = H_n(x+y)$$

for the case  $n = 2$ , where  $\binom{n}{k} = \frac{n!}{(n-k)!k!}$  is the binomial coefficient.

Workshop 2

1. Find the following limits

(a)  $\lim_{x \rightarrow \infty} \frac{x+1}{x+2}$ , (b)  $\lim_{x \rightarrow \infty} \frac{x^3+1}{2x^3+8x}$ , (c)  $\lim_{x \rightarrow 0} \frac{\sin^{-1} x}{x}$ ,  
(d)  $\lim_{x \rightarrow \infty} ((x+1)^{1/2} - x^{1/2})$ , (e)  $\lim_{x \rightarrow 0} \frac{\sin^{-1} x}{(\sin x)^2}$ .

2. Find the points of discontinuity of the following functions

(a)  $\frac{x^3+4x+6}{x^2-x-6}$ , (b)  $\sec x$ , (c)  $\frac{\sin x}{\sqrt{x}}$

3. Find  $\frac{dy}{dx}$  where  $y$  is given by:

(a)  $x^3e^{2x}$ , (b)  $\tanh x$ , (c)  $\sin(\ln(1+x^2))$ ,  
(d)  $\sin(\cos x)$ , (e)  $\exp(\sin^2 x)$ , (f)  $\sin^{-1}(x/(1+x))$ .

4. Find  $\frac{dy}{dx}$  in terms of  $x$  where  $y$  is given by

((a)  $y^3x + y + 7x^4 = 4$ , (b)  $y^y = x$ . (Hint : take logarithms.)

5.  $y = \cos^{-1} x$ ,  $|x| < 1$ . Show that  $\frac{dy}{dx} = -\frac{1}{\sqrt{1-x^2}}$ .

6. How many times is the function

$$f(x) = \begin{cases} (x-1)^2, & x \leq 1 \\ (x-1)^3, & x \geq 1 \end{cases}$$

differentiable at  $x = 1$ ?

7. Sketch the graph of the function

$$f(x) = \begin{cases} e^{-x} + x, & x \geq 0 \\ e^x, & x < 0 \end{cases}$$

Sketch the first, second and third derivatives of  $f(x)$ . How many times can  $f(x)$  be differentiated?

8. Find the Maclaurin series of  $(1-x)^{\frac{1}{4}}$  as far as  $x^3$  and use this series to find

$$\lim_{x \rightarrow 0} \frac{1 - (1-x)^{\frac{1}{4}}}{x}$$

9. Find the Taylor series expansion of  $\frac{1}{x}$  about  $x = 2$  as far as  $(x-2)^3$ .

10. Show that  $y = x \cos x - 3 \sin x + 2x$  has a stationary point at  $x = 0$  and that  $\frac{d^2y}{dx^2} = 0$  at  $x = 0$ . By using the Maclaurin series determine the nature of this stationary point and sketch the curve near to  $x = 0$ .

**Workshop 3**

**Graph sketching and integration**

1. Sketch the graph of  $y = x^3 - 6x^2 + 8x$ , listing the stationary points (determining their type) and the points of intersection with the axes.

2. Find the asymptotes of  $y = \frac{x-2}{2x+3}$  and sketch the graph.

3. Determine the asymptotes of the curve  $y = \frac{x}{3} + \frac{4}{x+1} + \frac{1}{x-2}$  and also the points of intersection of the curve with the line  $y = \frac{x}{3}$ . Use this information to sketch the graph.

4. Evaluate the following integrals:

(a)  $\int \frac{x+1}{x^2-3x+2} dx$

(b)  $\int \frac{2x^2+x+3}{x+1} dx$

(c)  $\int_3^8 \frac{1}{1+\sqrt{x+1}} dx$

(d)  $\int \frac{1}{x \ln x} dx$

(e)  $\int_2^4 x \ln x dx$

(f)  $\int x^2 \cosh x dx$

5. Calculate the total area made up of the two separate areas enclosed between the curve  $y = x^3 - 6x^2 + 8x$  (see question 1) and the  $x$ -axis between  $x = 0$  and  $x = 4$ .

Workshop 4

Partial Derivatives

1. Find  $\frac{\partial f}{\partial x}$ ,  $\frac{\partial f}{\partial y}$ ,  $\frac{\partial^2 f}{\partial x^2}$ ,  $\frac{\partial^2 f}{\partial x \partial y}$ ,  $\frac{\partial^2 f}{\partial y^2}$  where  $f(x, y) = 3x^2y - 4y^3 + 2e^x$ .

2. Show that  $w(x, t) = \sin(x - ct)$ ,  $c$  constant, is a solution of the wave equation

$$\frac{\partial^2 w}{\partial t^2} - c^2 \frac{\partial^2 w}{\partial x^2} = 0.$$

3.  $z(x, y)$  is defined implicitly by the equation  $yz - \ln(z) = x + y$ . Find  $\frac{\partial z}{\partial x}$  and  $\frac{\partial z}{\partial y}$ .

4. The cosine rule  $a^2 = b^2 + c^2 - 2bc \cos(A)$  gives  $A(a, b, c)$  implicitly in terms of  $a$ ,  $b$  and  $c$ . Calculate  $\frac{\partial A}{\partial a}$  and  $\frac{\partial A}{\partial b}$ .

5. Given  $f = f(x, y)$  and that  $x = e^u \cos v$ ,  $y = e^u \sin v$ , use the chain rule to find  $\frac{\partial f}{\partial u}$  and  $\frac{\partial f}{\partial v}$  in terms of  $x$ ,  $y$ ,  $\frac{\partial f}{\partial x}$  and  $\frac{\partial f}{\partial y}$ .

6.  $V = V(r, h)$  where  $r$  and  $h$  are functions of time  $t$ . Use the chain rule to find an expression for  $\frac{dV}{dt}$  in terms of  $\frac{dr}{dt}$  and  $\frac{dh}{dt}$ .

$V = \pi r^2 h / 3$  is the volume of a cone, base radius  $r$  and height  $h$ . If  $r$  and  $h$  are increasing at rates  $2 \text{ cm s}^{-1}$  and  $3 \text{ cm s}^{-1}$  respectively, show that, when  $r = 5$  and  $h = 15$ ,  $V$  is increasing at the rate  $125\pi \text{ cm}^3 \text{ s}^{-1}$ .

Workshop 5

Differential equations and probability

1. Find the the general solution of the differential equations:

$$(a) \quad \frac{dy}{dx} = \frac{3x^2}{y}, \quad (b) \quad y^2 \frac{dy}{dx} = e^x, \quad (c) \quad \frac{dy}{dx} = y(y-1).$$

2. Find the general solution of the differential equations:

$$(a) \quad \frac{dy}{dx} + \frac{2y}{x} = 2 \cos x, \quad (b) \quad \frac{dy}{dx} - \frac{y}{x^2} = \frac{4}{x^2}, \quad (c) \quad \frac{dy}{dx} + (2 \tan x)y = \sin x.$$

3. Solve the initial value problems:

$$(a) \quad \frac{dy}{dx} = \frac{y+2}{x-3}; y(0) = 1, \quad (b) \quad \frac{dy}{dx} + 3y = e^{-3x}; y(-1) = 2e^3.$$

4. Solve the equations:

$$(a) \quad x(x+1) \frac{dy}{dx} - (x+2)y = x^3(2x-3), \quad (b) \quad (x^2+3x+2) \frac{dy}{dx} + xy = x(x+1),$$
$$(c) \quad \frac{dy}{dx} + y \ln x = \exp(-x \ln x).$$



## Extra practice questions

	<b>Modern Engineering Mathematics</b> Glyn James (4th Edition)	<b>The Chemistry Maths book</b> Erich Steiner (2nd Edition)
Limits/ differentiation/ curve sketching	Exercises 8.26 (pg 548), 8.38 (pg 574) 8.3.11 (pg 581), 8.3.13 (pg 586) 8.4.2 (pg 596), 8.5.2 (pg 609)	Sections 4.4–4.10 (pg 122)
Partial fractions	Exercises 2.5.2 (pg 122)	Exercises 2.9, 17–22 (pg 59)
Maclaurin Series	Exercises 9.44 8, 12–17 (pg 702)	Section 7.7 (pg 224)
Integration	Exercises 8.76 (pg 625), 8.82 (pg 636) 8.84 (pg 640), 8.86 (pg 645) 8.13, 1–15 (pg 671)	Exercises 5.9 (pg 160), 6.8 (pg 187)
Multi variable calculus	Exercises 9.6.4 39–40, 42–46 (pg 724), Examples 9.22–9.24 (pg 718) Exercise 9.6.6 (pg 729)	Sections 9.1, 9.3, 9.6 (pg 289)
O.D.E.s	Exercises 10.5.4 (pg 788), 10.5.6 (pg 791), 10.5.8 (pg 794), 10.5.11 (pg 801)	Exercises 11.8 (pg 334)
Probability	Examples 13.28–13.29 (pg 1020), Exercise 13.8, 1 (pg 1035)	Section 21.10, 26 (pg 626)

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**Mathematics for Chemistry 1**  
Examples Class 1

1. The van der Waals equation for a slightly imperfect gas is

$$\left(p + \frac{n^2 a}{V^2}\right)(V - nb) - nRT = 0,$$

where  $p$  is the pressure,  $V$  is the volume,  $T$  is the temperature,  $n$  is the number of moles,  $R$  is the (universal) gas constant, and  $a$  and  $b$  are two constants.

(i) Express  $T$  and  $p$  as explicit functions of other variables. (ii) Rearrange the van der Waals equation and derive a cubic equation for  $V$ .

2. The virial equation of state of a gas can be approximated at low pressure as

$$pV_m = RT \left(1 + \frac{B}{V_m}\right),$$

where  $p$  is the pressure,  $V_m$  is the molar volume,  $T$  is the temperature,  $R$  is the gas constant, and  $B$  is the second virial coefficient. Express  $B$  as an explicit function of the other variables.

3. Kohlrausch's law for the molar conductivity  $\Lambda_m$  of a strong electrolyte at low concentration  $c$  is

$$\Lambda_m = \Lambda_m^0 - K\sqrt{c},$$

where  $\Lambda_m^0$  is the molar conductivity at infinite dilution and  $K$  is a constant. Express  $c$  as an explicit function of  $\Lambda_m$ .

4. For a system composed of  $N$  identical molecules, the Boltzmann distribution

$$\frac{n_i}{N} = e^{-\epsilon_i/kT}$$

gives the average fraction of molecules in the molecular state  $i$  with energy  $\epsilon_i$ .

(i) Show that the ratio  $n_i/n_j$  of the populations of states  $i$  and  $j$  depends only on the difference in energy of the two states. (ii) What is the ratio for two states with the same energy (degenerate states)?

5. The barometric formula

$$p = p_0 e^{-Mgh/RT}$$

gives the pressure of a gas of molar mass  $M$  at altitude  $h$ , when  $p_0$  is the pressure at sea level. Express  $h$  in terms of the other variables.

6. The chemical potential of a gas at pressure  $p$  and temperature  $T$  is

$$\mu = \mu_0 + RT \ln \frac{f}{p_0},$$

where  $f = \gamma p$  is the fugacity and  $\gamma$  is the fugacity coefficient. Express  $p$  as an explicit function of the other variables.

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**Mathematics for Chemistry 1**  
Examples Class 2

1. Find  $\frac{dV}{dp}$  at constant  $T$  and  $n$  for the following equations of state (assume that  $B$ ,  $a$ , and  $b$  are constants):

$$(a) \quad pV = nRT \left( 1 + \frac{nB}{V} \right), \quad (b) \quad \left( p + \frac{n^2a}{V^2} \right) (V - nb) = nRT.$$

2. The Lennard-Jones potential for the interaction of two molecules separated by distance  $R$  is

$$U(R) = \frac{A}{R^{12}} - \frac{B}{R^6},$$

where  $A$  and  $B$  are constants. The equilibrium separation  $R_e$  is that value of  $R$  at which  $U(R)$  is a minimum and the binding energy is  $D_e = -U(R_e)$ . Express (i)  $A$  and  $B$  in terms of  $R_e$  and  $D_e$ , (ii)  $U(R)$  in terms of  $R$ ,  $R_e$  and  $D_e$ .

3. The probability that a molecule of mass  $m$  in a gas at temperature  $T$  has speed  $v$  is given by the Maxwell-Boltzmann distribution

$$f(v) = 4\pi \left( \frac{m}{2\pi kT} \right)^{3/2} v^2 e^{-mv^2/2kT},$$

where  $k$  is Boltzmann's constant. Find the most probable speed (for which  $f(v)$  is a maximum).

4. The concentration of species  $B$  in the rate process  $A \xrightarrow{k_1} B \xrightarrow{k_2} C$ , consisting of two consecutive irreversible first-order reactions, is given by (when  $k_1 \neq k_2$ )

$$[B] = [A]_0 \frac{k_1}{k_2 - k_1} (e^{-k_1 t} - e^{-k_2 t})$$

(i) Find the time  $t$ , in terms of the rate constants  $k_1$  and  $k_2$ , at which  $B$  has its maximum concentration, and (ii) show that the maximum concentration is

$$[B]_{max} = [A]_0 \left( \frac{k_1}{k_2} \right)^{k_2/(k_2 - k_1)}$$

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**Mathematics for Chemistry 1**  
Examples Class 3

1. Explain how  $K$  and  $\Lambda_m^0$  in Kohlrausch's law

$$\Lambda_m = \Lambda_m^0 - K\sqrt{c}$$

can be obtained graphically from the results of measurements of  $\Lambda_m$  over a range of concentration  $c$ .

2. The Debye equation

$$\frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{\rho N_A}{3M\epsilon_0} \left( \alpha + \frac{\mu^2}{3kT} \right),$$

relates the relative permittivity (dielectric constant)  $\epsilon_r$  of a pure substance to the dipole moment  $\mu$  and polarizability  $\alpha$  of the constituent molecules, where  $\rho$  is the density at temperature  $T$ , and  $M$ ,  $N_A$ ,  $k$ , and  $\epsilon_0$  are constants. Explain how  $\mu$  and  $\alpha$  can be obtained graphically from the results of measurements of  $\epsilon_r$  and  $\rho$  over range of temperatures.

3. The Clausius-Clapeyron equation for liquid-vapour equilibrium is

$$\frac{d \ln p}{dT} = \frac{\Delta H_{vap}}{RT^2}.$$

If the enthalpy of vaporization,  $\Delta H_{vap}$ , is constant in the temperature range  $T_1$  to  $T_2$  show, by integrating both sides of the equation with respect to  $T$ , that

$$\ln \left( \frac{p_2}{p_1} \right) = \frac{\Delta H_{vap}}{R} \left( \frac{1}{T_1} - \frac{1}{T_2} \right)$$

4. The equation of state of a gas can be expressed in terms of the series

$$pV = nRT \sum_{i=0}^{\infty} B_i(T) \left( \frac{n}{V} \right),$$

where  $B_i$  are called virial coefficients. Find the first three coefficients for the van der Waals equation

$$\left( p + \frac{n^2 a}{V^2} \right) (V - nb) = nRT.$$

5. The energy density of black-body radiation at temperature  $T$  is given by the Planck formula

$$\rho(\lambda) = \frac{8\pi hc}{\lambda^5} [e^{hc/\lambda kT} - 1]^{-1},$$

where  $\lambda$  is the wavelength. Show that the formula reduces to the classical Rayleigh-Jeans law  $\rho = 8\pi kT/\lambda^4$  for long wavelengths ( $\lambda \rightarrow \infty$ ).

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**Mathematics for Chemistry 1**  
Examples Class 4

1. For the van der Waals equation

$$\left(p + \frac{n^2 a}{V^2}\right)(V - nb) - nRT = 0$$

Find (i)  $\left(\frac{\partial V}{\partial T}\right)_{p,n}$ , (ii)  $\left(\frac{\partial p}{\partial T}\right)_{V,n}$ .

2. If  $U = U(V, T)$  and  $p = p(V, T)$  are functions of  $V$  and  $T$  and if  $H = U + pV$ , show that

$$\left(\frac{\partial H}{\partial T}\right)_p - \left(\frac{\partial U}{\partial T}\right)_V = \left[\left(\frac{\partial U}{\partial V}\right)_T + p\right] \left(\frac{\partial V}{\partial T}\right)_p.$$

3. Given the total differential  $dG = -SdT + Vdp$ , show that  $\left(\frac{\partial S}{\partial p}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_p$

4. The rate equation of the second-order process  $A + B \xrightarrow{k} C$  is

$$-\frac{d(a-x)}{dt} = k(a-x)(b-x),$$

where  $[A] = a - x$ ,  $[B] = b - x$  are the concentrations of  $A$  and  $B$ , respectively, at time  $t$  and  $[A]_0 = a$ ,  $[B]_0 = b$  are initial concentrations. Derive the integrated rate equation assuming that  $a \neq b$ .

5. The system of two consecutive first-order processes  $A \xrightarrow{k_1} B \xrightarrow{k_2} C$  is modelled by the set of equations

$$\frac{d(a-x)}{dt} = -k_1(a-x), \quad \frac{dy}{dt} = k_1(a-x) - k_2y$$

where  $a - x$ ,  $y$  are the amounts of  $A$  and  $B$ , respectively, at time  $t$ . Given the initial conditions  $x = y = 0$  at  $t = 0$ , find the amount of  $A$  and  $B$  as functions of  $t$ . Assume that  $k_1 \neq k_2$ .