University of Huddersfield
School of Computing & Engineering

Week # 9   NFM2106/NFE2105

1. Solve the following ODEs of the form $dy/dx = F(x)$:

   (a) $\dfrac{dy}{dx} = 2x^4$;

   (b) $\dfrac{dy}{dx} - 2x^3 = e^{3x}$;

   (c) $2\dfrac{dy}{dx} + \cos x = 0$ subject to $y = 2.5$ when $x = \pi/2$;

   (d) $\dfrac{1}{2e^x} + 4 = x - 3\dfrac{dy}{dx}$ subject to $y(0) = 1/6$.

2. Solve the following ODEs of the form $dy/dx = F(y)$:

   (a) $\dfrac{dy}{dx} = 3 + 2y$;

   (b) $2\dfrac{dy}{dx} + 3y = 4$;

   (c) $\sqrt{y}\dfrac{dy}{dx} - 1 = 0$ subject to $y = 4$ when $x = 1/3$.

3. Solve the following equations by the method of variable separation:

   (a) $\dfrac{dy}{dx} = \dfrac{3x^2}{y}$;

   (b) $y^2\dfrac{dy}{dx} = e^x$;

   (c) $\dfrac{dy}{dx} = y(y - 1)$.

4. By finding a suitable integrating factor, solve the following inhomogeneous linear ODEs:

   (a) $\dfrac{dy}{dx} + \dfrac{2y}{x} = 2\cos x$;

   (b) $\dfrac{dy}{dx} - \dfrac{y}{x^2} = \dfrac{4}{x^2}$;

   (c) $\dfrac{dy}{dx} + (2\tan x)y = \sin x$;

   (d) $\dfrac{dy}{dx} + y = \dfrac{1}{1 + e^x}$;

   (e) $(x^3 + x)\dfrac{dy}{dx} + 4x^2y = 2$;

(f) $\dfrac{dy}{dx} + e^x y = e^x$;

(g) $\dfrac{dy}{dx} - \dfrac{5y}{x} = x^2$.

5. Solve the initial-value problems:

   (a) $\dfrac{dy}{dx} = \dfrac{y+2}{x-3}$, subject to $y(0) = 1$;

   (b) $\dfrac{dy}{dx} + 3y = e^{-3x}$, subject to $y(-1) = 2e^3$.

6. Find $y = y(x)$ that satisfies the following ODEs:

   (a) $y'' - 3y' + 2y = 0$;

   (b) $y'' + 4y = 0$;

   (c) $y'' + 5y' + 6y = 0$;

   (d) $y'' + y' + 2y = 0$,

   where the 'dash' stands for the usual derivative with respect to x.

7. Solve the following initial-value problems:

   (a) $\ddot{x} - 3\dot{x} + 2x = 0$, subject to $x(0) = 1$ and $\dot{x}(0) = 0$.

   (b) $\ddot{x} - 2\dot{x} + x = 0$, subject to $x(0) = 1$ and $\dot{x}(0) = 0$.

   (c) $\ddot{x} - 4\dot{x} + 20x = 0$, subject to $x(\pi/2) = 0$ and $\dot{x}(\pi/2) = 1$,

   where $x = x(t)$ and the 'dot' stands for the usual derivative with respect to t.

8. Show that the general solution of $\dfrac{d^2 y}{dx^2} - \lambda^2 y = 0$, where λ is real, may be written in terms of hyperbolic functions as

$$y(x) = C_1 \cosh(\lambda x) + C_2 \sinh(\lambda x),$$

   where C_1 and C_2 are arbitrary real constants.

ANSWERS:

In all the answers recorded below C, C_1 and C_2 are arbitrary real constants.

1. (a) $y = \dfrac{2}{5}x^5 + C$; (b) $y = \dfrac{1}{2}x^4 + \dfrac{1}{3}e^{3x} + C$; (c) $y = -\dfrac{1}{2}\sin x + 3$;

   (d) $y = \dfrac{x^2}{6} + \dfrac{1}{6}e^{-x} - \dfrac{4x}{3}$.

2. (a) $y = Ce^{2x} - \dfrac{3}{2}$; (b) $\dfrac{1}{3}\ln|4 - 3y| + \dfrac{x}{2} = C$; (c) $\dfrac{2}{3}y^{3/2} = x + 5$.

3. (a) $\dfrac{y^2}{2} = x^3 + C$; (b) $\dfrac{y^3}{3} = e^x + C$; (c) $\ln\left|\dfrac{y-1}{y}\right| = x + C$.

4. (a) $y = 2\sin x + \dfrac{4\cos x}{x} - \dfrac{4\sin x}{x^2} + \dfrac{C}{x^2}$; (b) $y = -4 + Ce^{-1/x}$;

   (c) $y = \cos x + C\cos^2 x$; (d) $y = e^{-x}[\ln(1 + e^x) + C]$;

   (e) $y = \dfrac{x^2 + C}{(x^2 + 1)^2} + \dfrac{2\ln x}{(x^2 + 1)^2}$; (f) $y = 1 + Ce^{-e^x}$; (g) $y = -\dfrac{1}{2}x^3 + Cx^5$.

5. (a) $\ln|y + 2| = \ln|x - 3|$; (b) $y = e^{-3x}(x + 3)$.

6. (a) $y = C_1 e^x + C_2 e^{2x}$; (b) $y = C_1\cos(2x) + C_2\sin(2x)$; (c) $y = C_1 e^{3x} + C_2 e^{2x}$;

   (d) $y = e^{-x/2}\left[C_1\cos\left(\dfrac{x\sqrt{7}}{2}\right) + C_2\sin\left(\dfrac{x\sqrt{7}}{2}\right)\right]$.

7. (a) $y = 2e^t - e^{2t}$; (b) $y = (1 - t)e^t$; (c) $y = \dfrac{1}{4}e^{2t-\pi}\sin(4t)$.