Week # 8 NFM2106/NFE2105

1. Solve for X when

$$A = \begin{bmatrix} 6 & 5 \\ -1 & 3 \\ 3 & 1 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 & 3 \\ 2 & 0 \\ -4 & 1 \end{bmatrix}.$$

   (a) $2X + 3A = B$;
   (b) $2A + 4B = -2X$;
   (c) $6X - 4A - 3B = O$;
   (d) $3X + 2A = 5B$.

2. Use the following matrices to compute the indicated expression if it is defined

$$A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}, \qquad B = \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix}, \qquad C = \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix},$$

$$D = \begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix}, \qquad E = \begin{bmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{bmatrix},$$

   (a) $D + E$; (b) $4E - 2D$; (c) $-3(D + 2E)$;
   (d) AB; (e) ABC; (f) CC^T;
   (g) $(DA)^T$; (h) $(2D^T - E)A$; (i) $(B^T - B)C$.

3. Find all values of k, if any, that satisfy

$$\begin{bmatrix} k & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 2 \\ 0 & 2 & -3 \end{bmatrix} \begin{bmatrix} k \\ 1 \\ 1 \end{bmatrix} = 0.$$

   Repeat the question for

$$\begin{bmatrix} 2 & 2 & k \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 3 \\ 0 & 3 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ k \end{bmatrix} = 0.$$

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4. Evaluate the determinants of the following matrices:

$$A = \begin{bmatrix} 3 & 5 \\ -2 & 4 \end{bmatrix}; \qquad B = \begin{bmatrix} 4 & 1 \\ 8 & 3 \end{bmatrix}; \qquad C = \begin{bmatrix} -5 & 7 \\ -7 & 3 \end{bmatrix}; \qquad D = \begin{bmatrix} \sqrt{2} & \sqrt{6} \\ 4 & \sqrt{3} \end{bmatrix}.$$

5. Evaluate the determinants of the following matrices:

$$A = \begin{bmatrix} a+b & a & a \\ a & a+b & a \\ a & a & a+b \end{bmatrix}; \qquad B = \begin{bmatrix} a & b & 1 \\ 2 & 3 & 1 \\ 0 & -1 & 1 \end{bmatrix}; \qquad C = \begin{bmatrix} k+1 & k-1 & 7 \\ 2 & k-3 & 4 \\ 5 & k+1 & k \end{bmatrix}.$$

6. Let $A = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$.
   (a) Show that $A^2 - 2A + 5I_2 = O_2$.
   (b) Show that $A^{-1} = \frac{1}{5}(2I - A)$.

7. Find the values of p for which the following matrices are singular:

$$(a) \begin{bmatrix} p & 2 & 0 \\ 0 & p+1 & 2 \\ 0 & 1 & p \end{bmatrix}; \qquad (b) \begin{bmatrix} p & 0 & 1 \\ 0 & p & 3 \\ 2 & 2 & p-2 \end{bmatrix}.$$

8. Show that the matrix $A = \begin{bmatrix} \sec\theta & \tan\theta \\ \tan\theta & \sec\theta \end{bmatrix}$ is invertible, and find its inverse.

9. Show that the matrix

$$A = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

   is invertible for all values of θ, and then find A^{-1}.

10. For each of the matrices included below you should:
    (a) find the determinant using the cofactor expansion formula;
    (b) find the adjoint;
    (c) hence, or otherwise, identify the inverse matrix (if it exists).

$$(a) \begin{bmatrix} -3 & 2 & 1 \\ 4 & 5 & 6 \\ 2 & -3 & 1 \end{bmatrix}; \qquad (b) \begin{bmatrix} -3 & 4 & 2 \\ 6 & 3 & 1 \\ 4 & -7 & -8 \end{bmatrix}.$$

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11. Solve the following systems of simultaneous equations by using Cramer's rule:

(a)

$$3x + 3y + 5z = 1$$
$$3x + 5y + 9z = 2$$
$$5x + 9y + 17z = 4$$

(b)

$$4x - y - z = 1$$
$$2x + 2y + 3z = 10$$
$$5x - 2y - 2z = -1$$

(c)

$$2x + y = 4$$
$$-2x + 3y + z = -11$$
$$x + 3y + z = -2$$

(d)

$$2x - y - 2z = 1$$
$$x + y + 3z = 6$$
$$-2x + 3z = -1$$

Additional practice questions can be found in the recommended textbook.

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ANSWERS:

1.

$$\text{(a) } X = \begin{bmatrix} -9 & -6 \\ 2.5 & -4.5 \\ -6.5 & -1 \end{bmatrix}; \qquad \text{(b) } X = \begin{bmatrix} -6 & -11 \\ -3 & -3 \\ 5 & -3 \end{bmatrix};$$

$$\text{(c) } X = \begin{bmatrix} 4 & 29/6 \\ 1/3 & 2 \\ 0 & 7/6 \end{bmatrix}; \qquad \text{(d) } X = \begin{bmatrix} -4 & 5/3 \\ 4 & -2 \\ -26/3 & 1 \end{bmatrix}.$$

2.

$$\text{(a) } D+E = \begin{bmatrix} 7 & 6 & 5 \\ -2 & 1 & 3 \\ 7 & 3 & 7 \end{bmatrix}; \qquad \text{(b) } 4E-2D = \begin{bmatrix} 22 & -6 & 8 \\ -2 & 4 & 6 \\ 10 & 0 & 4 \end{bmatrix};$$

$$\text{(c) } -3(D+2E) = \begin{bmatrix} -39 & -21 & -24 \\ 9 & -6 & -15 \\ -33 & -12 & -30 \end{bmatrix}; \qquad \text{(d) } AB = \begin{bmatrix} 12 & -3 \\ -4 & 5 \\ 4 & 1 \end{bmatrix};$$

$$\text{(e) } ABC = \begin{bmatrix} 3 & 45 & 9 \\ 11 & -11 & 17 \\ 7 & 17 & 13 \end{bmatrix}; \qquad \text{(f) } CC^T = \begin{bmatrix} 21 & 17 \\ 17 & 35 \end{bmatrix};$$

$$\text{(g) } (DA)^T = \begin{bmatrix} 0 & -2 & 11 \\ 12 & 1 & 8 \end{bmatrix}; \qquad \text{(h) } (2D^T - E)A = \begin{bmatrix} -6 & -3 \\ 36 & 0 \\ 4 & 7 \end{bmatrix};$$

$$\text{(h) } (B^T - B)C = \begin{bmatrix} 3 & 1 & 5 \\ -1 & -4 & -2 \end{bmatrix};$$

3. (i) $k = -1$; (ii) $k = -2$ or $k = -10$.

4. $|A| = 22$; $|B| = 4$; $|C| = 34$; $|D| = -3\sqrt{6}$.

5. $|A| = b^2(3a+b)$; $|B| = 2(2a-b-1)$; $|C| = k^3 - 8k^2 - 10k + 95$.

6. –

7. (a) $p = 0$ or $p = -2$ or $p = 1$; (b) $p = 0$ or $p = -2$ pr $p = 4$.

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8.
$$A^{-1} = \begin{bmatrix} \sec\theta & -\tan\theta \\ -\tan\theta & \sec\theta \end{bmatrix}.$$

9.
$$A^{-1} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

10. (a)
$$A^{-1} = \frac{1}{75}\begin{bmatrix} -23 & 5 & -7 \\ -8 & 5 & -22 \\ 22 & 5 & 23 \end{bmatrix}; \qquad adj(A) = \begin{bmatrix} 23 & -5 & 7 \\ 8 & -5 & 22 \\ -22 & -5 & -23 \end{bmatrix};$$

(b)
$$A^{-1} = \frac{1}{151}\begin{bmatrix} -17 & 18 & -2 \\ 52 & 16 & 15 \\ -54 & -5 & -33 \end{bmatrix}; \qquad adj(A) = \begin{bmatrix} -17 & 18 & -2 \\ 52 & 16 & 15 \\ -54 & -5 & -33 \end{bmatrix}.$$

11. (a) $x = 0$, $y = -1/2$, $z = 1/2$; (b) $x = 1$, $y = 1$, $z = 2$; (c) $x = 3$, $y = -2$, $z = 1$; (d) $x = 2$, $y = 1$, $z = 1$.