

Week #7

NFM2106/NFE2105

1. Simplify the following:

(a) j^3 ; (b) j^4 ; (c) j^5 ; (d) j^{15} ; (e) j^{22} .

2. If $z_1 = 2 - 5j$, $z_2 = 1 + 7j$, and $z_3 = -3 - 4j$, determine the following in the form $a + bj$, where a and b are real numbers that you must specify:

- (a) $z_1 - z_2 + z_3$;
- (b) $2z_1 + z_2 - z_3$;
- (c) $z_1 - (4z_2 - z_3)$;
- (d) z_1/z_2 ;
- (e) z_2/z_3 ;
- (f) z_3/z_1 .

3. Determine the values of the real numbers x and y such that

$$(3x - 5y) + j(x + 3y) = 20 + 2j.$$

4. Determine the real and imaginary parts of the expression

$$(1 - 3j)^2 + (2 + 5j)j - \frac{3(4 - j)}{1 - j}.$$

5. If $z = x + yj$ and $\bar{z} = x - yj$ are conjugate complex numbers, determine the values of the real numbers x and y such that

$$4z\bar{z} - 3(z - \bar{z}) = 2 + j.$$

6. Determine the **modulus** (r in decimals, where appropriate, correct to three significant figures), and the **argument** (θ in degrees, correct to the nearest degree) of the following complex numbers:

(a) $1 - j$; (b) $-3 + 4j$; (c) $-\sqrt{2} - \sqrt{2}j$; (d) $\frac{1}{2} - \frac{\sqrt{3}}{2}j$; (e) $-7 - 9j$.

7. If $z = 4 - 5j$, verify that jz has the same modulus as z , but that the value of the principal argument of jz is greater by 90° than the value of the principal argument of z .

8. Illustrate the following operations on the Argand diagram:

(a) $(6 - 11j) + (5 + 3j) = 11 - 8j$;

(b) $(6 - 11j) - (5 + 3j) = 1 - 14j$.

9. In the following cases,

(a) express the complex numbers z_1 and z_2 in the polar form $r\angle\theta$ using only the principal value of θ (i.e. the principal argument);

(b) determine the product z_1z_2 and the quotient $\frac{z_1}{z_2}$, in polar form, using only the principal value of the argument;

(i) $z_1 = 1 + j$ and $z_2 = \sqrt{3} - j$;

(ii) $z_1 = -\sqrt{2} - \sqrt{2}j$ and $z_2 = -3 - 4j$;

(iii) $z_1 = -4 - 5j$ and $z_2 = 7 - 9j$.

10. Determine the following in the form $a + bj$, expressing the real numbers a and b in decimal form, correct to four significant figures:

(a) $(1 + j\sqrt{3})^{10}$; (b) $(2 - 5j)^{-4}$.

11. Determine the fifth roots of the complex number $(-4 + 4j)$ in the form $a + bj$, expressing the real numbers a and b in decimal form, where appropriate, correct to two decimal places.

ANSWERS:

1. (a) $-j$; (b) 1 ; (c) j ; (d) $-j$; (e) -1 .
2. (a) $-2 - 16j$; (b) $8 + j$; (c) $-5 - 37j$; (d) $-0.66 - 0.38j$;
(e) $-1.24 - 0.68j$; (f) $0.48 - 0.79j$.
3. $x = 5, y = -1$.
4. real part = -20.5 ; imaginary part = -8.5 .
5. $x = \pm\sqrt{17}/6, y = -1/6$.
6. (a) $r = 1.41, \theta = -45^\circ$; (b) $r = 5, \theta = 127^\circ$; (c) $r = 2, \theta = -135^\circ$;
(d) $r = 1, \theta = -60^\circ$; (e) $r = 11.4, \theta = -128^\circ$.
7. if $z_1 = 4 - 5j$, then $|z_1| = \sqrt{41}, \text{Arg}(z_1) = -51^\circ$; if $z_2 = j(4 - 5j)$, then
 $|z_2| = |j||z_1| = \sqrt{41}, \text{Arg}(z_2) = -51^\circ + 90^\circ$.
8. Construct the sum and the difference of the two complex numbers.
9.
 - (i) $z_1 = \sqrt{2}\angle 45^\circ, z_2 = 2\angle -30^\circ, z_1z_2 = 2\sqrt{2}\angle 15^\circ, \frac{z_1}{z_2} = \frac{1}{\sqrt{2}}\angle 75^\circ$;
 - (ii) $z_1 = 2\angle -135^\circ, z_2 = 5\angle -127^\circ, z_1z_2 = 10\angle 98^\circ, \frac{z_1}{z_2} = \frac{2}{5}\angle -8^\circ$;
 - (iii) $z_1 = 6.40\angle -128.66^\circ, z_2 = 11.40\angle -52.125^\circ, z_1z_2 = 72.96\angle 179.215^\circ, \frac{z_1}{z_2} = 0.56\angle -76.53^\circ$.
10. (a) $-512.0 - 886.8j$; (b) $0.00005809 - 0.001188j$.
11. $1.26 + 0.64j, -0.22 + 1.40j, -1.40 + 0.22j, -0.64 - 1.26j, 1.00 - 1.00j$.