Week #6b                                                                NFM2106/NFE2105

1.  (a) If $a = 2i - 3j + 5k$ and $b = 3i + j - 2k$, evaluate $|(a + b) \cdot (a - b)|$.

    (b) If $a = 2i - j + k$ and $b = i + 2j - 3k$, find the magnitude of the vector
    $(2a + b) \times (a - 2b)$.

2.  In which of the following cases are the vectors $a$ and $b$ orthogonal

    (i).      $a = i + j + k$,              $b = -i + j + k$?

    (ii).     $a = 3i + j + 2k$,            $b = -i + j + k$?

3.  For what value of $\lambda \in \mathbb{R}$ are the vectors $a$ and $b$ parallel

    (i).    $a = 3i + \lambda j$,      $b = 9i - 12j$?

    (ii).   $a = \lambda i - 8j$,      $b = 7i - 10j$?

    (ii).   $a = 2i + \lambda j - 2k$,      $b = i - 2j + \lambda k$?

4.  A force $R$ acts on a particle, where $R = (7i + 16j)$ N. Calculate:

    (a) the magnitude of $R$ (2dp);

    (b) the angle between the line of action of $R$ and $i$, giving your answer to the nearest
    degree;

    (c) The force $R$ is the resultant of two forces $P$ and $Q$. The line of action of $P$ is
    parallel to the vector $i + 4j$, and the line of action of $Q$ is parallel to the vector
    $i + j$. Determine the forces $P$ and $Q$ expressing each in terms of $i$ and $j$.

5.  The force $R$ is given by the vector $6i + 2j$, where the units of force are newtons. This
    force is the resultant of a force $P$ parallel to $i$ and a force $Q$ parallel to $i + j$. Find the
    magnitudes of $P$ and $Q$.

6.  (optional) If $u$ and $v$ are arbitrary vectors, then show that

    (i).    $|u - v|^2 + |u + v|^2 = 2(|u|^2 + |v|^2)$;

    (ii).   $u \cdot v = \dfrac{1}{4}(|u + v|^2 - |u - v|^2)$.

7.  If vectors $a + b$ and $a - b$ are perpendicular, show that $a$ and $b$ are of equal magnitude.

8.  If the vectors $a$, $b$, $c$ are represented by the sides $\overrightarrow{BC}$, $\overrightarrow{CA}$, $\overrightarrow{AB}$ of a triangle $ABC$, show
    that $b \times c = c \times a = a \times b$.

9. If $a$, $b$, $c$ are arbitrary vectors, verify that $(c - a) \times (b - a) = (b - c) \times (a - c)$.

10. Given that $a$, $b$ are both *unit vectors*, and that the magnitude of the force $F$ is 10:
    (*i*) find the components of $F$ in the directions of $i$ and $j$, respectively;
    (*ii*) find the components of $F$ in the direction of $a$ and $b$ (see the sketch included below).
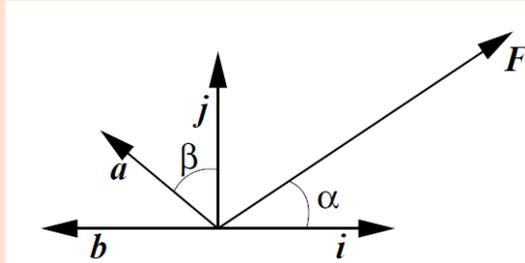


11. Find the acute angle between the vectors

$$u = 3i - j + 2k \qquad \text{and} \qquad v = -i - 4j + 4k \,.$$

12. The vertices $A$, $B$, $C$ of a triangle $ABC$ have coordinates $(3, -1, 4)$, $(2, 1, 0)$, and $(-1, 2, 1)$, respectively. Find the angle $BAC$.

ANSWERS:

1. (a) 24; (b) $25\sqrt{3}$.

2. (i) not orthogonal; (ii) orthogonal.

3. (i) $\lambda = -4$; (ii) $\lambda = 28/5$; (iii) $\lambda = \pm 2$.

4. (a) $|\boldsymbol{R}| \simeq 17.46$; (b) $\theta \simeq 66°$; (c) $\boldsymbol{P} = 3(\boldsymbol{i} + 4\boldsymbol{j})$, $\boldsymbol{Q} = 4(\boldsymbol{i} + \boldsymbol{j})$.

5. (a) $|\boldsymbol{P}| = 4$, $|\boldsymbol{Q}| = 2\sqrt{2}$.

6. Use the fact that $\boldsymbol{w} \cdot \boldsymbol{w} = |\boldsymbol{w}|^2$ for any vector $\boldsymbol{w}$.

7. –

8. Use that $\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c} = \boldsymbol{0}$.

9. Expand LHS and RHS separately, then compare results.

10. (i) $10\cos$   and $10\sin$  , respectively;
    (ii) $10\sin($  –  $)$ and $-10\cos$  , respectively.

11. $\theta \simeq 65.25°$.

12. $\angle BAC \simeq 34.6°$.