Week #6a   NFM2106/NFE2105

1. Using the Taylor expansion for $\ln(1 + x)$, show that

$$\ln x = \ln a + \frac{1}{a}(x - a) - \frac{1}{2a^2}(x - a)^2 + \frac{1}{3a^3}(x - a)^3 - \dots , \qquad \text{for } 0 < x \leq 2a ,$$

   where $a > 0$ is a given number.

2. Expand $f(x)$ as indicated and specify the values of x for which the expansion is valid:

   (a) $f(x) = 3x^3 - 2x^2 + 4x + 1$ in powers of $(x - 1)$;
   (b) $f(x) = (1 + x)^{-1}$ in powers of $(x - 1)$;
   (c) $f(x) = e^{-4x}$ in powers of $(x + 1)$;
   (d) $f(x) = \sin x$ in powers of $(x - \pi)$.

3. Use the first three non-zero terms of the appropriate Taylor polynomials to estimate the following numerical values:

$$(a) \ \sin 10°; \qquad (b) \ e^{0.8}; \qquad (c) \ \sqrt{e}; \qquad (d) \ \cos 6° .$$

   In each case, compare the value found with that given by your calculator.

4. (a) Show that

$$\ln\left(\frac{1 + x}{1 - x}\right) = 2\left(x + \frac{x^3}{3} + \frac{x^5}{5} + \dots\right) , \qquad -1 < x < 1 .$$

   (b) Setting $x = \frac{1}{3}$ in this formula, use the first three non-zero terms to estimate $\ln 2$.
   Use a calculator to check the accuracy of your result.

5. The field strength H of a magnet in the shape of a cylindrical bar at a point on its axis at distance x from its centre is given by the formula

$$H = \frac{M}{2\ell}\left[\frac{1}{(x - \ell)^2} - \frac{1}{(x + \ell)^2}\right] ,$$

   where 2ℓ is the length of the magnet and M is its moment. Show that if ℓ is very small compared with x then $H \simeq 2M/x^3$.

6. By expanding e^{-x^2} as a Maclaurin series, show that

$$\int_0^{1/2} e^{-x^2} \, dx \simeq 0.461 .$$