

Week # 5

NFM2106/NFE2105

1. Use the substitution  $u = x + 3$  in order to determine the indefinite integral

$$\int x\sqrt{x+3} dx$$

2. Use the substitution  $u = x^2 - 1$  in order to evaluate the integral

$$\int_1^5 x\sqrt{x^2-1} dx$$

3. Use an appropriate substitution to find the value of the integral

$$\int_1^2 x\sqrt{4x^2-3} dx.$$

4. Evaluate between the give limits:

(a)  $\int_{\pi}^{2\pi} (x+2) \sin x dx$ ;      (e)  $\int_0^{\ln 2} e^{2x+5} dx$ ;

(b)  $\int_0^{\pi/2} x \sin(2x) dx$ ;      (f)  $\int_{\pi/8}^{\pi/2} \cos^2(4x) dx$ ;

(c)  $\int_0^{\pi/6} (2x+1) \cos(3x) dx$ ;      (g)  $\int_1^{\ln 3} xe^{3x} dx$ ;

(d)  $\int_0^{\pi/3} \sin^2(3x) dx$ ;      (h)  $\int_1^3 \frac{x+1}{x+2} dx$ .

5. Use the trapezium rule to evaluate the following integrals, giving all answers correct to 3 decimal places:

(i)  $\int_0^1 \sqrt{1+x^2} dx$       using 5 intervals;

(ii)  $\int_{0.2}^{0.6} \frac{dx}{x}$       using 4 intervals;

(iii)  $\int_{\pi/6}^{\pi/2} \frac{\sin x}{x} dx$       using 2 intervals.

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6. Use Simpson's rule to find the following:

(i)  $\int_0^1 \sqrt{4+x^4} dx$       using 4 intervals; work to 3 d.p., rounding your answer to 2 d.p. ;

(ii)  $\int_0^{0.6} \frac{dx}{\sqrt{1+x^4}}$       using 6 intervals; work to 4 d.p., rounding your answer to 3 d.p. ;

(iii)  $\int_0^1 \ln(1+e^x) dx$       using 8 intervals; work to 5 d.p., rounding your answer to 4 d.p. .

7. (a) Calculate the area between the curve  $y = \cos x$ ,  $-\pi < x < \pi$ , and the line  $y = \frac{1}{2}$ ;  
(b) Find the area between the line  $y = x$  and the curve  $y = x^2$ ;  
(c) Find the area contained between the two curves  $y = 3x - x^2$  and  $y = x + x^2$ .

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ANSWERS:

1.  $\frac{2}{5}(x-2)(x+3)^{3/2}$ .

2.  $16\sqrt{6}$ .

3.  $\frac{1}{12}(13\sqrt{13}-1)$

4. (a)  $-3\pi-4$ ; (b)  $\frac{\pi}{4}$ ; (c)  $\frac{\pi+1}{9}$ ;

(d)  $\frac{\pi}{6}$ ; (e)  $\frac{3e^5}{2}$ ; (f)  $\frac{3\pi}{16}$ ;

(g)  $-3-\frac{2e^3}{9}+9\ln 3$ ; (h)  $2-\ln\frac{5}{3}$ .

5. (a) 1.150; (b) 1.117;

(c) 0.850.

6. (a) 2.05; (b) 0.593;

(c) 0.9838.

7. (a)  $\sqrt{3}-\frac{\pi}{3}$ ; (b)  $\frac{1}{6}$ ; (c)  $\frac{1}{3}$ .