University of Huddersfield
School of Computing & Engineering

Week # 4 $\hspace{6cm}$ NFM2106/NFE2105

1. Find the coordinates of the minima and maxima of the following functions:

   (a) $f_1(x) = x^3 - 3x^2 - 9x + 2$;

   (b) $f_2(x) = x^4 - 2x^2 - 3$;

   (c) $f_3(x) = x(x^2 - 1)^2$;

   (d) $f_4(x) = xe^{-x^2}$;

   (e) $f_5(x) = x^2(x - 1)^2$.

2. It costs a publisher 10,000 GBP to set up the presses for the print run of a book and 8 GBP to cover the material costs for each book printed. In addition, machinery servicing, labour, and warehousing add another $6.25 \times 10^{-7}x^2$ GBP to the cost of each book if x copies are manufactured during the printing.
   How many copies should the manufacturer print in order to maximise the average cost per book?

3. Yohance owns a bicyle shop. She must pay the manufacturer 75 GBP for each bicycle. Market research suggests that if Yohance charges her customers x GBP per bicycle, she can expect to sell $N(x) = 4.5 \times 10^6/x^2$ of them. What price should she charge to maximise her profit, and how many bicycles should she order from the manufacturer?

4. (Integration by substitution) Find the following integrals:

   (a) $\displaystyle \int \frac{4x + 6}{x^2 + 3x - 7}\,dx$;
   $\qquad$ (e) $\displaystyle \int \frac{\cos x}{5 + \sin x}\,dx$;

   (b) $\displaystyle \int \frac{4x^2}{x^3 - 27}\,dx$;
   $\qquad$ (f) $\displaystyle \int \frac{x}{x^2 + 19}\,dx$;

   (c) $\displaystyle \int \frac{x}{(2 - x)^{2/3}}\,dx$;
   $\qquad$ (g) $\displaystyle \int xe^{-(x^2+4)}\,dx$;

   (d) $\displaystyle \int 3x\sqrt{4 - x^2}\,dx$;
   $\qquad$ (h) $\displaystyle \int 5x \sin(x^2 - 9)\,dx$.

5. Find the following integrals:

   (a) $\displaystyle \int \frac{3x + 2}{x + 4}\,dx$;
   $\qquad$ (b) $\displaystyle \int \frac{2x - 5}{x + 3}\,dx$;
   $\qquad$ (c) $\displaystyle \int \frac{5x + 11}{7x + 4}\,dx$.

6. (Integration by parts) Find the following indefinite integrals:

(a) $\displaystyle\int 3x\mathrm{e}^{5x}\,\mathrm{d}x$;

(b) $\displaystyle\int x^2\mathrm{e}^{-x}\,\mathrm{d}x$;

(c) $\displaystyle\int \mathrm{e}^{3x}\sin x\,\mathrm{d}x$;

(d) $\displaystyle\int x^2\mathrm{e}^{3x}\,\mathrm{d}x$.

ANSWERS:

The **integration constants** in Q.4, 5, 6 are omitted for the sake of brevity; you are expected to mention this constant in the class tests, etc. Failure to do so will result in loss of marks.

1. (a) $x = -1$ (max) and $x = 3$ (min);

    (b) $x = \pm 1$ (min) and $x = 0$ (max);

    (c) $x = -1$ (max), $x = 1$ (min), $x = -5^{-1/2}$ (min), $x = 5^{-1/2}$ (max);

    (d) $x = -2^{-1/2}$ (min), $x = 2^{-1/2}$ (max);

    (e) $x = 2$ (min), $x = 1/2$ (max), $x = 1$ (min).

2. $2,000$ books.

3. 150 GBP per bicycle; $N(150) = 200$.

4. (a) $2\ln(x^2 + 3x - 7)$; (b) $\dfrac{4}{3}\ln(x^3 - 27)$; (c) $-\dfrac{3}{4}(x + 6)(2 - x)^{1/3}$;

    (d) $-(4 - x^2)^{3/2}$; (e) $\ln(\sin x + 5)$; (f) $\ln\sqrt{x^2 + 19}$;

    (g) $-\dfrac{1}{2}e^{-x^2 - 4}$; (h) $-\dfrac{5}{2}\cos(9 - x^2)$.

5. (a) $3x - 10\ln(x + 4)$; (b) $2x - 11\ln(x + 3)$;

    (c) $\dfrac{1}{49}(35x + 57\ln(7x + 4))$.

6. (a) $\dfrac{3}{25}e^{5x}(5x - 1)$; (b) $-e^{-x}(x^2 + 2x + 2)$;

    (c) $-\dfrac{1}{10}e^{3x}(\cos x - 3\sin x)$; (d) $\dfrac{1}{27}e^{3x}(9x^2 - 6x + 2)$.