

Week # 3

NFM2106/NFE2105

1. For each of the functions included below, find dy/dx at $x = 2$:

(a) $y = x^4 - 3x^3 + \sqrt{x+1} - \frac{1}{2\sqrt{x}}$;

(b) $y = e^{x^2+x}$;

(c) $y = (3x + 2)e^{-x}$;

(d) $y = e^x/(1 + e^x)$.

2. Differentiate with respect to x (simplifying the results as much as possible):

(a) $\ln(2x + 3)$;

(b) $\ln(x^2 + 2x + 3)$;

(c) $\ln[(2x + 1)/(1 - 3x)]$;

(d) $\frac{\ln(x + 1)}{x}$;

(e) $x^3 \cosh(x)$;

(f) $e^{-3x} \sinh(3x)$;

(g) $\sqrt{\sin x}$;

(h) $\cos^3(x)/\cos(3x)$;

(k) $\sin^4(x) \cos(4x)$.

3. Find d^2y/dx^2 for the following functions (simplifying the results as much as possible):

(a) $y = x^3\sqrt{1 + x^2}$;

(b) $y = \ln(x^2 + x + 1)$;

(c) $y = \frac{\ln x}{x}$;

(d) $y = e^{-3x} \sin(2x)$.

4. For each of the following trigonometric equations, state the solutions in the indicated range:

(a) $\sin(2x + \pi/3) = -1/2$, with $-\pi \leq x \leq \pi$;

(b) $\sin(3x + 4\pi/3) = -1/\sqrt{2}$, with $-\pi/2 \leq x \leq \pi$;

(c) $\sin(x) + \cos(2x) = 0$, with $-\pi < x < \pi/4$;

(d) $\sin(x + \pi/4) = -\sqrt{3}/2$, with $0 < x < 3\pi$;

(e) $\sin(2x + 2\pi/3) = -1$, with $\pi < x < 2\pi$.

5. Find the gradient of the graph of the function

$$y = \frac{11 - 4x}{3 - x}$$

at the point $(2, 3)$.

6. Find the gradient of the graph of the function

$$y = \frac{x + 1}{\sqrt{x}}$$

at the point $(1, 2)$.

ANSWERS:

1. (a) $-4 + \frac{1}{8\sqrt{2}} + \frac{1}{2\sqrt{3}}$; (b) $5e^6$; (c) $-5e^{-2}$; (d) $\frac{e^2}{(1 + e^2)^2}$.

2. (a) $\frac{2}{3 + 2x}$; (b) $\frac{2(1 + x)}{3 + 2x + x^2}$; (c) $\frac{-5}{6x^2 + x - 1}$; (d) $\frac{1}{x(x + 1)} - \frac{\ln(x + 1)}{x^2}$;

(e) $3x^2 \cosh x + x^3 \sinh x$; (f) $3e^{-6x}$; (g) $\frac{\cos x}{2\sqrt{\sin x}}$;

(h) $\frac{3 \cos^2 x \sin(2x)}{\cos^2(3x)}$; (k) $4 \cos(5x) \sin^3(x)$.

3. (a) $\frac{x(12x^4 + 19x^2 + 6)}{(1 + x^2)^{3/2}}$; (b) $\frac{1 - 2x - 2x^2}{(1 + x + x^2)^2}$;

(c) $\frac{2 \ln x - 3}{x^3}$; (d) $e^{-3x} [5 \sin(2x) - 12 \cos(2x)]$.

4. (a) $-\frac{7\pi}{12}, -\frac{\pi}{4}, \frac{5\pi}{12}, \frac{3\pi}{4}$;

(b) $-\frac{\pi}{36}, \frac{5\pi}{36}, \frac{23\pi}{36}, \frac{29\pi}{36}$;

(c) $-\frac{5\pi}{6}, -\frac{\pi}{6}$;

(d) $\frac{13\pi}{12}, \frac{17\pi}{12}$;

(e) $\frac{17\pi}{12}$.

5. dy/dx at $x = 2$ is -1 .

6. dy/dx at $x = 1$ is 0 .