Week # 2 NFM2106/NFE2105

1. Find the quotient and the remainder obtained from dividing $P(x)$ by $Q(x)$, where:
   (a) $P(x) = x^4 - 3x^2 + 6x - 11$ and $Q(x) = x + 2$;
   (b) $P(x) = 2x^5 + 5x^2 - 9x + 17$ and $Q(x) = x^2 + x + 3$;
   (c) $P(x) = 3x^5 + 4x^4 - 9x^2 + 14x - 13$ and $Q(x) = 3x^2 + 6x + 4$;
   (d) $P(x) = x^4 + 21x^2 - 16x + 19$ and $Q(x) = x^2 - 3$.

2. Find the partial fraction decomposition for the following rational functions:
   (a) $\dfrac{3x+1}{(x-2)(x-5)}$;
   (b) $\dfrac{5x+6}{(x-2)^2}$;
   (c) $\dfrac{2x-1}{(x^2+5)(x-1)}$;
   (d) $\dfrac{3x-8}{x^2-4}$.

3. Simplify the following expressions (without using a calculator!):
   (a) $2\ln\left(\dfrac{2}{3}\right) - \ln\left(\dfrac{8}{9}\right)$;
   (b) $25^{\log_5 3}$;
   (c) $\log_a(b^2) + \log_{a^2}(b^4)$;
   (d) $\log_3 5 \cdot \log_4 9 \cdot \log_5 2$;
   (e) e^y, where $y = \dfrac{1}{2}\ln\left(\dfrac{1-x}{1+x}\right)$;
   (f) $\ln y$, where $y = \dfrac{(x^2+1)^{3/2}}{(x^4+1)^{1/3}(x^4+4)^{1/5}}$.

4. Solve the following logarithmic equations (leave your answers in surd form, if applicable):
   (a) $\log_7 x - \log_7(2x-5) = \log_7 2 - \log_7(x-3)$;
   (b) $\log_3(x-5) + 2\log_3\sqrt{x-3} = 2$;
   (c) $\log_2(x-3) + \log_2(x-2) = 2$;

September 2021
Dr. Ciprian D. Coman
C.Coman@hud.ac.uk

5. The questions that follow require you to establish an **identity**. This is usually done by evaluating the LHS and RHS separately, and then comparing the results. Make sure you watch the relevant video-example on Brightspace (which contains the solutions for (a) and (c)).

   (a) $(1+\tan^2\theta)(1-\sin^2\theta)=1$;

   (b) $(1+\tan\theta)^2+(1+\cot\theta)^2=(sec\theta+\text{cosec}\theta)^2$;

   (c) $\dfrac{\sin 2\theta}{1+\cos 2\theta}=\tan\theta$;

   (d) $\dfrac{\cos 2\theta}{1+\sin 2\theta}=\tan\left(\dfrac{\pi}{4}-\theta\right)$;

   (e) $\dfrac{1-\cos 2\theta+\sin 2\theta}{1+\cos 2\theta+\sin 2\theta}=\tan\theta$;

   (f) $(2\cos\theta+1)(2\cos\theta-1)=2\cos 2\theta+1$.

   [**This question is not examinable**, it is meant to provide you with an opportunity to practice some of the trigonometry introduced this week.]

6. Consider the polynomial $P(x)=3x^6-12x^4+3x^3-5x^2+12x+5$. Determine whether the following statements are 'true' or 'false':

   (a) $P(x)$ has a root between $a=1.0$ and $b=1.5$;

   (b) $P(x)$ has a root between $a=-1.0$ and $b=0.0$;

   (c) $P(x)$ has a root between $a=1.6$ and $b=2.0$.

Dr. Ciprian D. Coman
C.Coman@hud.ac.uk

ANSWERS:

1. Notation: $q(x)$ =quotient; $r(x)$ =remainder;
   (a) $q(x) = x^3 - 2x^2 + x + 4$, $r(x) = -19$;
   (b) $q(x) = 2x^3 - 2x^2 - 4x + 15$, $r(x) = -12x - 28$;
   (c) $q(x) = x^3 - \dfrac{2x^2}{3} - \dfrac{19}{9}$, $r(x) = \dfrac{80x}{3} - \dfrac{41}{9}$;
   (d) $q(x) = x^2 + 24$, $r(x) = -16x + 91$.
2. (a) $\dfrac{16/3}{x-5} - \dfrac{7/3}{x-2}$;
   (b) $\dfrac{16}{(x-2)^2} + \dfrac{5}{x-2}$;
   (c) $\dfrac{1}{6}\left[\dfrac{1}{x-1} - \dfrac{x-11}{x^2+5}\right]$;
   (d) $-\dfrac{1}{2}\left[\dfrac{1}{x-2} - \dfrac{7}{x+2}\right]$.
3. (a) $-\ln(2) \equiv \ln\left(\dfrac{1}{2}\right)$; (b) 9; (c) $\dfrac{4\ln(b)}{\ln(a)} \equiv 4\log_a b$; (d) 1; (e) $\dfrac{\sqrt{1-x^2}}{1+x}$;
   (f) $\dfrac{3}{2}\ln(x^2+1) - \dfrac{1}{3}\ln(x^4+1) - \dfrac{1}{5}\ln(x^4+4)$.
4. (a) $x = 5$; (b) $x = 4 + \sqrt{10}$; (c) $x = \dfrac{1}{2}(5 + \sqrt{17})$.
5. –
6. (a) true; (b) true; (c) true.

Dr. Ciprian D. Coman
C.Coman@hud.ac.uk