

Week # 10

NFM2106/NFE2105

1. Use the table of Laplace transforms, together with the linearity property, to obtain the transforms of the following functions:

(a) $y = 3t^2 + 5t - 7$;

(b) $y = (t + 9)^2$;

(c) $y = t + \frac{2t}{3} - 6e^{4t} + 3\sin(8t)$;

(d) $y = 3\cos(6t) - 6\sin(6t)$;

(e) $y = e^{4t} \cosh(5t)$;

(f) $y = (\sin t - \cos t)^2$.

2. Obtain the inverse Laplace transforms of the following functions:

(a) $Y(s) = \frac{1}{s - 2}$;

(b) $Y(s) = \frac{5}{s^3} - \frac{2}{s^2} + \frac{1}{s}$;

(c) $Y(s) = \frac{8}{s} - \frac{4}{3s^2} + \frac{6}{s^2 + 49}$.

3. Resolve into partial fractions and hence obtain the inverse Laplace transforms of the following functions:

(a) $Y(s) = \frac{2s + 1}{s(s + 1)}$;

(b) $Y(s) = \frac{5s - 10}{s^2 - s - 6}$;

(c) $Y(s) = \frac{3s + 1}{s^2 + 2s - 3}$;

(d) $Y(s) = \frac{s - 1}{(s + 2)^2}$;

(e) $Y(s) = \frac{5s^2 + 2}{s(s - 1)(s + 2)}$;

(f) $Y(s) = \frac{4s + 1}{s(s^2 + 9)}$.

4. Re-arrange into the form

$$c_1 \left(\frac{s}{s^2 + a^2} \right) + c_2 \left(\frac{a}{s^2 + a^2} \right),$$

and hence obtain the inverse Laplace transforms of:

(a) $Y(s) = \frac{s + 6}{s^2 + 49}$;

(b) $Y(s) = \frac{3s - 1}{s^2 + 25}$;

(c) $Y(s) = \frac{2s - 5}{4s^2 + 25}$;

5. Complete the square of the first two terms in the denominator and re-arrange into the form

$$c_1 \left\{ \frac{s + k}{(s + k)^2 + a^2} \right\} + c_2 \left\{ \frac{a}{(s + k)^2 + a^2} \right\} .$$

Hence, obtain the inverse Laplace transforms of:

(a) $Y(s) = \frac{s - 4}{s^2 - 4s + 20}$;

(b) $Y(s) = \frac{2s + 5}{s^2 + 6s + 10}$;

(c) $Y(s) = \frac{s + 4}{s^2 + 6s + 130}$;

(d) $Y(s) = \frac{s + 7}{s^2 + 8s + 23}$.

6. Use the Laplace transform to solve the following first-order linear IVPs:

(a) $\frac{dy}{dt} - 2y = 3t$, subject to $y(0) = 1$;

(b) $\frac{dy}{dt} + 5y = 1 - 2t$, subject to $y(0) = -1$;

(c) $\frac{dy}{dt} + 4y = \sin(t)$, subject to $y(0) = 4$.

7. Use the Laplace transform to solve the following second-order linear IVPs:

(a) $y'' + 2y = 1$, subject to $y(0) = 1$ and $y'(0) = 1$;

(b) $y'' + 4y = 6t$, subject to $y(0) = 0$ and $y'(0) = 1$;

(c) $y'' + 6y' + 10y = 4e^{-t/2}$, subject to $y(0) = 0$ and $y'(0) = 0$;

(d) $y'' + y = t$, subject to $y(0) = 1$ and $y'(0) = -2$;

(e) $y'' - 3y' + 2y = 4e^{2t}$, subject to $y(0) = -3$ and $y'(0) = 5$,

where $y = y(t)$ and the 'dash' stands for the usual derivative with respect to t .

ANSWERS:

1. (a) $6s^{-3} + 5s^{-2} - 7s^{-1}$; (b) $2s^{-3} + 18s^{-2} + 81s^{-1}$;

(c) $\frac{5}{3s^2} - \frac{6}{s-4} + \frac{24}{s^2+64}$; (d) $\frac{3(s-12)}{s^2+36}$;

(e) $\frac{s-4}{(s+1)(s-9)}$; (f) $\frac{1}{s} - \frac{2}{s^2+4}$.

2. (a) e^{2t} ; (b) $\frac{5t^2}{2} - 2t + 1$; (c) $8 - \frac{4t}{3} + \frac{6}{7}\sin(7t)$.

3. (a) $1 + e^{-t}$; (b) $4e^{-2t} + e^{3t}$; (c) $2e^{-3t} + e^t$;

(d) $e^{-2t}(1-3t)$; (e) $\frac{11}{3}e^{-2t} + \frac{7}{3}e^t - 1$; (f) $\frac{1}{9}[1 - \cos(3t) + 12\sin(3t)]$.

4. (a) $\cos(7t) + \frac{6}{7}\sin(7t)$; (b) $3\cos(5t) - \frac{1}{5}\sin(5t)$; (c) $\frac{1}{2}\left[\cos\left(\frac{5t}{2}\right) - \sin\left(\frac{5t}{2}\right)\right]$.

5. (a) $\frac{1}{2}e^{2t}[2\cos(4t) - \sin(4t)]$; (b) $e^{-3t}(2\cos t - \sin t)$;

(c) $\frac{1}{11}e^{-3t}[11\cos(11t) + \sin(11t)]$; (d) $e^{-4t}\left[\cos(t\sqrt{7}) + \frac{3}{\sqrt{7}}\sin(t\sqrt{7})\right]$.

6. (a) $\frac{1}{4}(7e^{2t} - 6t - 3)$; (b) $\frac{1}{25}(7 - 2t - 32e^{-5t})$;

(c) $\frac{69}{17}e^{-4t} + \frac{1}{17}(4\sin t - \cos t)$.

7. (a) $\frac{1}{2} + \frac{1}{2}\cos(t\sqrt{2}) + \frac{1}{\sqrt{2}}\sin(t\sqrt{2})$; (b) $\frac{3t}{2} - \frac{1}{4}\sin(2t)$;

(c) $-\frac{8}{29}e^{-3t}(2\cos t + 5\sin t - 2e^{5t/2})$; (d) $t + \cos t - 3\sin t$; (e) $e^t(-7 + 4e^t + 4te^t)$.