Mathematics 2L — Linear Modelling

Problems 3

1. Use the graphical approach and then the simplex method to maximise $P = 3x_1 + 5x_2$ over all non-negative x_1, x_2 satisfying

$$x_1 + x_2 \leqslant 50, \quad 2x_1 + x_2 \leqslant 110, \quad 2x_1 + 9x_2 \leqslant 360, \quad 3x_1 + x_2 \leqslant 120.$$

2. Use the graphical method and the simplex method to maximise $P = 2x_1 + 5x_2$ over all non-negative x_1, x_2 satisfying

$$x_1 \leqslant 8, \quad 3x_1 + 2x_2 \leqslant 32, \quad x_1 + 3x_2 \leqslant 24, \quad x_1 + 2x_2 \leqslant 18.$$

3. Sketch roughly the 3-dimensional region

$$0 \leqslant x_1 \leqslant 3, \quad 0 \leqslant x_2 \leqslant 3, \quad 0 \leqslant x_3 \leqslant 3, \quad 2x_1 + x_2 + 2x_3 \geqslant 4,$$

and determine the maximum and minimum values of the following functions in this region:

(i) $P = -4x_1 - 2x_2 - x_3$, (ii) $Q = x_1 + x_2 + x_3$, (iii) $R = x_1 - 4x_2 + x_3$.

4. A company manufactures two types of cordless telephones, the profit on the high security 'scrambler' model being 20% more than that of the less secure regular model. Both of these telephones are assembled from four main electronic components which are restocked monthly. In the current market, the company can sell all produce and so practices a manufacturing policy which maximizes monthly profits by making optimal use of the available stock. How many of each model should the company aim to produce given that the current stock situation is

Component	Number in stock	Number per scrambler model	Number per regular model
F320 Processor	24000	2	0
80286 Processor	48000	3	2
Superhet Diode	36000	1	3
Field-effect Transistor	54000	2	4

5. A farmer owns 600 hectares of arable land and is planning to grow root crops, leaf crops and also leave a portion of it uncultivated in order to attract an EU grant for taking agricultural land out of production. The following table estimates the time input (in person days) and profit margin (in £) per hectare:

Effort	Person Days per hectare	Profit per hectare
Root crops	4	2,500
Leaf crops	2	1,000
Fallow	0	500

She decides to grow at least 50 hectares of each crop, smaller quantities not being economic. In order to maintain elegibility for future EU benefits, a minimum of 75% of the available arable land must be cultivated. The total labour at her disposal amounts to 1400 person days. Solve this linear programming problem and determine the optimal farming strategy.

6. A coal producer manufactures Phurnicite for use in central heating boilers. This fuel is a mixture of Anthracite, Lignite and Coke and must meet statutory regulations on sulphur emission, namely that its sulphur content does not exceed 3%. Additionally, the ash residue of combusted fuel must not exceed 3.25%. Solve the linear programming problem connected with the production of Phurnacite given the following information:

Fuel	Sulphur Content (%)	Ash Content (%)	Cost per tonne (£)
Anthracite	2	3	90
Lignite	4	4	40
Coke	3	2	80