

Mathematics 2L — Linear Modelling

Problems 2

1. Use the following tables to calculate [to 4dp] the first and second derivatives of the function $f(x)$ at the midpoint of the specified intervals:

(i)

x	0.15	0.2	0.25
$f(x)$	1.1618	1.2214	1.2840

(ii)

x	1.1	1.2	1.3
$f(x)$	0.09531	0.1823	0.2624

Verify that these tables are consistent with the view that $f(x) = e^x$ and $f(x) = \log x$ respectively, and comment on the accuracy of your answers.

2. Use the method of finite differences to solve the boundary value problems

(i) $y'' + xy = 1$, $y(0) = 0$, $y(1) = 1$, (ii) $y'' + y = 0$, $y(0) = 1$, $y'(1) = 0$.

In each case take $h = 1/4$.

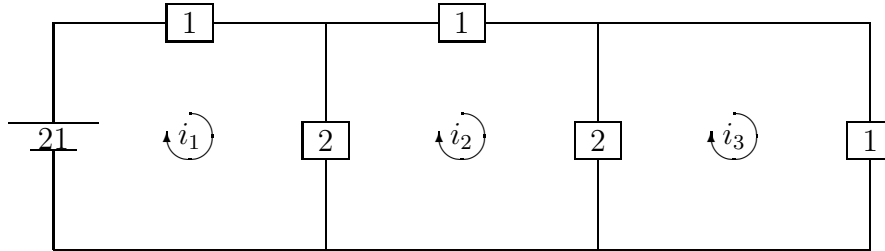
Show that $y(x) = \cos x + \tan 1 \sin x$ is the solution of (ii) and check your answer against it.

3. Use the method of finite differences with $h = 1/3$ to solve the following boundary value problems:

(i) $x^2y'' - xy' + y = x^2$, $y(1) = 0$, $y(2) = 4$,

(ii) $x^2y'' - xy' + y = x^2$, $y'(1) = 0$, $y(2) = 1$.

4. A known voltage, 21, is applied to the circuit shown.



What currents flow in the wires?

5. Find the straight line that provides the best fit to the data

x	1	2	3	4
y	0	1	1	2

in the least squares sense.

6. Find the straight line that provides the best fit to the data

x	0	1	2	4
y	0	0	1	5

in the least squares sense.

7. Find the quadratic $y = ax^2 + b$ which best fits the data

x	-1	0	1	2
y	3	1	2	5

in the least squares sense.

8. Find the plane $z = ax + by + c$ that best approximates the data

x	-1	0	1	2
y	-1	0	2	3
z	-0.4	0.3	1.6	2.5

in the least squares sense.

9. Two different theories suggest that variables x and y are related by either $y = ax^n$ or $y = ce^{kx}$. An experiment gives the data

x	1	2	3	4	5
y	1	3	7	15	30

By taking logarithms and using the least-squares procedure, find the best values for the constants a , n and c , k .

For each fit, calculate the sum of the squares of the errors. Which fit seems better?