

Mathematics 2L — Linear Modelling

Problems 1

1 Given the three matrices

$$A = \begin{bmatrix} 1 & 3 & 4 \\ -2 & 1 & 6 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 6 \\ 2 & -3 \\ -4 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 1 & -1 \\ 2 & 0 & -1 \\ 1 & 1 & 2 \end{bmatrix},$$

find which of the following products exist and evaluate those that do:

$$AB, \quad BA, \quad AC, \quad CA, \quad BC, \quad CB.$$

2 For the matrices

$$A = \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \quad C = AB = \begin{bmatrix} 1/2 & -1/2 \\ 1/2 & -1/2 \end{bmatrix},$$

find A^2 , A^3 , B^2 , B^3 , C^2 and C^3 .

3 Find *all* 2×2 real matrices that commute with $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$.

4 Find *all* the 2×2 real matrices A, B for which

$$(i) \ A^2 = -I, \quad (ii) \ B^2 = 0.$$

5 Use *eros* to find the inverse (when it exists) of each of the matrices:

$$(i) \ \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}, \quad (ii) \ \begin{bmatrix} 0 & 0 & 3 \\ -1 & 1 & 1 \\ 2 & -2 & 0 \end{bmatrix}, \quad (iii) \ \begin{bmatrix} 1 & 3 & 5 \\ 3 & 12 & 18 \\ 5 & 18 & 30 \end{bmatrix}.$$

6 Use Gaussian elimination to solve the systems of equations

$$\begin{array}{lll} (i) & 2x - 3y & = 3 \\ & 4x - 5y + z & = 7 \\ & 2x - y - 3z & = 5 \end{array} \quad \begin{array}{lll} (ii) & u + v + w & = 2 \\ & u + 3v + 3w & = 0 \\ & u + 3v + 5w & = 2 \end{array} \quad \begin{array}{lll} (iii) & a + 2b + 2c & = 0 \\ & 3a - 3b + 2c & = 5 \\ & 2a + 4b - c & = -5 \end{array}$$

7 Use Gaussian elimination to find all values of a for which

$$\begin{aligned}ax_1 + x_2 &= 1 \\4x_1 + ax_2 &= 2\end{aligned}$$

has no solutions.

8 Find the LU decomposition of the matrix

$$A = \begin{bmatrix} 1 & 2 & 10 \\ 2 & -8 & -1 \\ -1 & 3 & 2 \end{bmatrix}.$$

Use this decomposition to solve the system of equations

$$\begin{aligned}x + 2y + 10z &= 15 \\2x - 8y - z &= 24 \\-x + 3y + 2z &= -6\end{aligned}.$$

[Parts of L and U to find: $L = (2, -1, -5/12)$, $U = (-12, -21, 13/4)$, $\mathbf{z} = (15, -6, 13/2)^T$, $\mathbf{x} = (1, -3, 2)^T$.]

9 Write the system of equations

$$\begin{aligned}2x + y - z &= 1 \\-2x + 2y + 2z &= 2 \\6y + 4z &= 4\end{aligned}$$

in the form $A\mathbf{x} = \mathbf{b}$. Find the LU decomposition of A and *hence* solve the equations for \mathbf{x} . [Check your solutions.]

10 Repeat **8** and **9** using *partial pivoting*.