University of Glasgow

Department of Mathematics

Course 1T 2007–2008

Contents, Aims and Objectives

General aims of the course Mathematics–1T is intended to provide a useful and worthwhile half-year's Mathematics course leading on from the level reached in Mathematics–1R. It aims, in particular,

- to increase students' competence and confidence in handling mathematical ideas and notations that they may meet in further Mathematics courses and in other subjects,
- to develop students' ability to apply Mathematics to practical problems, and more generally to improve their problem-solving capabilities,
- to extend students' knowledge in calculus and algebra, introducing them to new topics like vectors and the study of differential equations.

Learning Objectives for the course as a whole From the point of view of a student, the general learning objectives in Mathematics–1T include:

- becoming familiar with all the ideas and practised in all the skills covered in the course, and, in particular, knowing and being able to use all mathematical results covered in the lectures and laboratories;
- being able to solve or work through problems involving one or more parts of the course contents;
- being able to present mathematical work in a coherent and literate way.

It should be noted that the aims and objectives of the course embrace the acquisition and/or development of important transferable skills. It is intended that students will enhance their ability to think logically and analytically, and will become better able to express their ideas in clear and coherent mathematical language.

Learning Objectives for the Calculus half of the course

1. Integration (about 8 lectures)

Aims To introduce integration, and various techniques used in applying it.

Contents

- Integration of various functions, as anti-differentiation
- Integration by change of variable, and by parts
- Partial fractions.

Objectives After this section students should be able to

- know and use the standard integrals
- use change of variable methods, with completing the square where necessary
- use trigonometric changes of variable
- use integration by parts
- use partial fractions, and integrate the resulting rational functions.

2. Definite Integrals (about 5 lectures)

Aims To introduce and apply the evaluation of definite integrals.

Contents

- Notation for and techniques for dealing with definite integrals,
- Areas under and between curves,
- Volumes of revolution,
- Arc length of a curve.

Objectives After this section students should

- understand the notation used for definite integrals
- be able to recognize and apply the appropriate method for evaluating various definite integrals
- know the connection between definite integrals and the evaluation of areas
- know and be able to apply the formulae to calculate volumes of revolution
- be able to find the length of an arc of a curve.

3. Approximation Methods (about 5 lectures)

Aims To study various methods of approximating in Mathematics.

Contents

- Simpson's Rule
- Newton's Method and the Intermediate Value Theorem
- Maclaurin Series.

Objectives After this section students should

- know and be able to use Simpson's Rule for approximating definite integrals
- know and be able to use Newton's Method for approximating the root of an equation
- be able to use the Intermediate Value Theorem to locate a root of an equation
- know the Maclaurin Series of various functions
- be able to find Maclaurin Series
- be able to manipulate Maclaurin Series appropriately
- know about intervals of convergence of Maclaurin Series.

4. Differential Equations (about 6 lectures)

Aims To develop techniques for solving certain differential equations.

Contents

- Classification of differential equations
- Solution of various first-order differential equations
- Solution of second-order differential equations with constant coefficients
- Applications of differential equations.

Objectives After this section students should

- know how to classify certain types of differential equations
- be able to solve first-order separable differential equations
- be able to solve first-order linear differential equations
- be able to solve second-order differential equations with constant coefficients
- be able to solve various problems involving the solution of differential equations.

Learning Objectives for the Algebra half of the course

1. Geometry and Vectors (about 8 lectures)

Aims To study vectors and their application to geometric problems, and to study lines and surfaces, all in 3-dimensional space.

Contents

- Definition of vector and the associated notation and terminology (e.g. magnitude, unit vector)
- Use of geometrical steps (directed line segments) to represent vectors
- Vector arithmetic
- Parallel vectors
- Position vectors
- Section formula
- Coordinates in 1, 2 and 3 dimensions
- Components of vectors
- Equations of surfaces (planes, spheres, cylinders and lines)
- Intersections of surfaces
- Scalar product and angles
- Vector product (component form only).

Objectives After this section students should

- be acquainted with the definition of vector and the associated notation and terminology
- know how to use geometrical steps (directed line segments) to represent vectors
- know the definitions of addition, subtraction and scalar multiplication of vectors, and the basic laws that they obey
- be able to identify vectors parallel to given vectors
- know the definition of position vector and be able to solve a variety of geometric problems using position vectors
- be able to prove the existence of the centroid of a triangle using position vectors
- be acquainted with coordinates and the distance formulae in 3 dimensions
- be able to express the position vector of a point in terms of the vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$
- know the definition of the components of a vector and be able to solve a variety of geometric problems using position vectors in component form
- be able to find the equations of planes, spheres, cylinders and lines, and to identify such surfaces from their equations
- be able to determine whether lines and planes intersect and find their intersection
- know the definition of scalar product and apply it to find angles between vectors, lines and surfaces
- state and prove a test for perpendicularity of vectors
- know the definition of vector product in component form and apply it to finding equations of planes.

2. Growth and Decay (about 2 lectures)

Aims To study the properties of exponential growth and decay.

Contents

- Definition of exponential growth and decay
- Rate of change, logarithmic graph, fractional and percentage change, half-life.

Objectives After this section students should

- know the definitions of exponential growth and decay
- know their properties, namely that the rate of change is proportional to the amount present, the logarithmic graph is a straight line, and the fractional and percentage changes depend only on the length of time elapsed
- know the definition of half-life, be able to prove the $\frac{\ln 2}{k}$ formula and know the $\frac{70}{r}$ rule
- be able to carry out calculations concerning quantities with exponential growth and decay.

3. Logic and Proof (about 4 lectures)

Aims To increase understanding of what constitutes a proof, and to introduce various methods of proof.

Contents

- "If P then Q" statements and their proofs
- Direct proof
- Counterexamples
- Proof by contradiction
- Proof by induction.

Objectives After this section students should

- know the difference between absolute statements and hypothetical ("if P then Q") statements
- know how to lay out the proof of a hypothetical statement
- know what a direct proof is, and understand the necessity of dealing with the most general case
- be able to understand the difference between a statement and its converse
- appreciate that a single counterexample is sufficient to demonstrate that a statement is false, and have gained some experience in identifying suitable counterexamples
- be able to understand proof by contradiction and use it in suitable cases, and in particular to prove that $\sqrt{2}$ is irrational
- be able to understand proof by induction and use it in suitable cases.

4. Counting, Binomial Theorem and Probability (about 10 lectures)

Aims To study the methods of counting, the binomial theorem and its applications, and the concept of probability.

Contents

- The relationship between OR, AND, NOT and $+$, $\times$, $-$
- The multiplication principle
- Permutations and combinations
- Pascal's triangle
- Binomial theorem, direct applications and applications to trigonometry and calculus
- Introduction to probability
- Examples on probability using permutations, combinations, complements and the hypergeometric formula
- Conditional probability, law of total probability, independence
- The binomial probability formula.

Objectives After this section students should

- know the relationship between OR, AND, NOT and $+$, $\times$, $-$, know the multiplication principle, and be able to apply that knowledge to solve various counting problems
- know the definitions of permutation and combination and appreciate the difference between them
- know formulae for counting permutations and combinations and be able to apply them to suitable examples
- know the Binomial Theorem, and be able to apply it to expansions of the form $(a+b)^n$ and also to re-express powers of cosines and sines in terms of cosines and sines of multiple angles, and vice-versa, and hence integrate certain trigonometric functions
- be able to construct Pascal's triangle and understand its connection with the Binomial Theorem
- know the notation and terminology used in discussing probability (e.g. experiment, outcome, events, equally likely outcomes, mutually exclusive events, relative frequency) and understand at an intuitive level what is meant by the probability of an event
- know the Fundamental Law of Probability, the meaning of $P(A \cup B)$, $P(A \cap B)$, $P(\overline{A})$, the associated arithmetic rules and the hypergeometric formula
- know the definition of the conditional probability $P(B|A)$, the difference between $P(B|A)$ and $P(A \cap B)$, the formula linking the two, the law of total probability, how to calculate $P(A|B)$ from $P(B|A)$, and the concept of independent events
- know the binomial probability formula
- be able to tackle a wide range of examples on probability using all the theory of counting and probability described above.