

Mathematics 1T (Algebra)

Summary of Week #2

- The medians of any triangle meet in a common point (a **median** is a line joining one vertex to the midpoint of the opposite side). The common point is the **centroid** of the triangle.

If the position vectors of the vertices of triangle ABC are, respectively, \mathbf{a} , \mathbf{b} , and \mathbf{c} , then the centroid has position vector given by

$$\frac{1}{3}(\mathbf{a} + \mathbf{b} + \mathbf{c}).$$

OBS. the proof of these facts is **examinable!**

- **Coordinates in 3D** (full details on *Handout #1*)

1. The position of points in three-dimensional space (3D) can be located with respect to some reference point (the *origin*) by using three numbers. One way to use three numbers to specify a point is by having them represent *signed distances* from the origin, measured in the direction of three mutually perpendicular lines passing through the origin. Such a set of lines is called a *Cartesian coordinate system*.

Each of the lines is referred to as a *coordinate axis*, and we call these axes the x -axis, the y -axis, and the z -axis. Usually, we regard the x - and y -axes as lying in a horizontal plane and the z -axis as vertical.

The plane that contains the x - and y -axis is called the x, y -plane. In a similar way we can define the x, z -plane and the y, z -plane.

2. With respect to a Cartesian coordinate system, the coordinates of a point P in 3D is represented by an ordered triple of scalars, (x_P, y_P, z_P) . The numbers x_P , y_P , and z_P are the signed distances of P from the origin, measured in the direction of the x -axis, the y -axis, and the z -axis, respectively.

- The distance between points $P(x_P, y_P, z_P)$ and $Q(x_Q, y_Q, z_Q)$ is

$$PQ = \sqrt{(x_P - x_Q)^2 + (y_P - y_Q)^2 + (z_P - z_Q)^2}$$

- The distance from $P(x_P, y_P, z_P)$ to the origin is given by

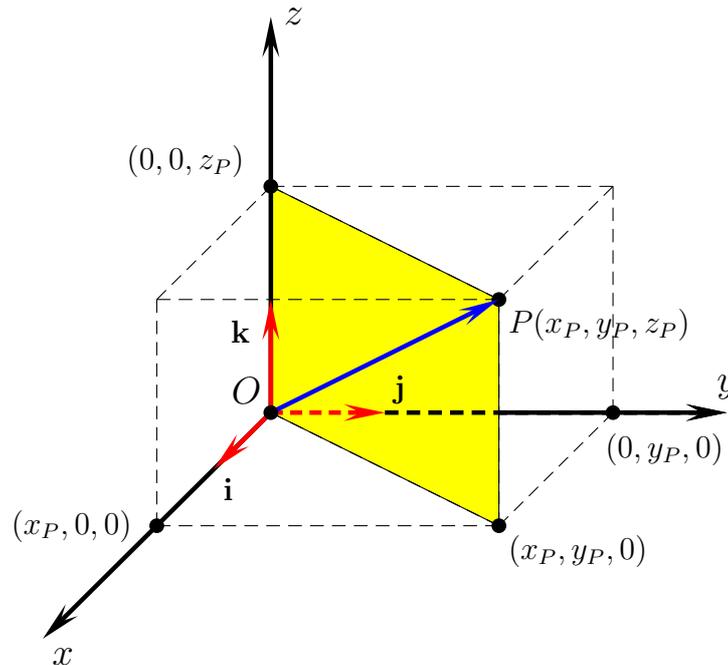
$$OP = \sqrt{x_P^2 + y_P^2 + z_P^2}$$

which can be obtained by taking $Q = O(0, 0, 0)$ in the above formula.

• **Vectors in 3D**

1. Given a Cartesian coordinate system in 3D, we let \mathbf{i} , \mathbf{j} , and \mathbf{k} denote the unit vectors in the direction of the positive x -, y -, and z -axes. We can think of these vectors as being represented by arrows from the origin to the points $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$, respectively.
2. If P has coordinates (x_P, y_P, z_P) then \mathbf{p} , the position vector of P , is given by

$$\mathbf{p} = x_P\mathbf{i} + y_P\mathbf{j} + z_P\mathbf{k} . \quad (1)$$



3. Every *vector* \mathbf{u} is the position vector of some *point*, say, (a, b, c) . So, by (1), we have

$$\mathbf{u} = a\mathbf{i} + b\mathbf{j} + c\mathbf{k} . \quad (2)$$

The numbers a, b, c in (2) are called the components of \mathbf{u} and we shall often use the alternative notation

$$\mathbf{u} = (a, b, c)$$

instead of (2).

4. **Very important fact:**

The *vector* (a, b, c) is the position vector of the *point* (a, b, c) .

5. If $\mathbf{u} = (a, b, c)$ then the *magnitude* of this vector is

$$|\mathbf{u}| = \sqrt{a^2 + b^2 + c^2}$$

• **Vector arithmetic in component form**

Let $\mathbf{u} = (u_1, u_2, u_3)$ and $\mathbf{v} = (v_1, v_2, v_3)$ be two arbitrary vectors, and $\lambda \in \mathbb{R}$ (a scalar).

1. The usual arithmetic operations ($\mathbf{u} + \mathbf{v}$, $\mathbf{u} - \mathbf{v}$, and $\lambda\mathbf{u}$) are defined component-wise,

$$(u_1, u_2, u_3) + (v_1, v_2, v_3) = (u_1 + v_1, u_2 + v_2, u_3 + v_3) \quad (3)$$

$$(u_1, u_2, u_3) - (v_1, v_2, v_3) = (u_1 - v_1, u_2 - v_2, u_3 - v_3) \quad (4)$$

$$\lambda(u_1, u_2, u_3) = (\lambda u_1, \lambda u_2, \lambda u_3) \quad (5)$$

2. An equality between two vectors in compact form

$$\mathbf{u} = \mathbf{v} \quad \text{or} \quad (u_1, u_2, u_3) = (v_1, v_2, v_3)$$

is equivalent to three scalar equations, namely

$$\begin{cases} u_1 = v_1, \\ u_2 = v_2, \\ u_3 = v_3. \end{cases}$$

• **Equations of simple surfaces** (see *Handout #2* for more details):

1. **planes:** We studied two simple cases

- planes parallel to the coordinate planes: these have equations of the form

$$x = K \quad (\text{parallel to the } y, z\text{-plane}),$$

$$y = K \quad (\text{parallel to the } x, z\text{-plane}),$$

$$z = K \quad (\text{parallel to the } x, y\text{-plane}),$$

where K is a given scalar.

- planes perpendicular to the coordinate planes: these have equations of the form

$$Ax + By = K \quad (\text{perpendicular to the } x, y\text{-plane}),$$

$$Ax + Bz = K \quad (\text{perpendicular to the } x, z\text{-plane}),$$

$$Ay + Bz = K \quad (\text{perpendicular to the } y, z\text{-plane}),$$

where A, B, K are given scalars.

2. **spheres:** There are two forms

- centre at the origin, $O(0, 0, 0)$,

$$x^2 + y^2 + z^2 = R^2,$$

where (x, y, z) are the coordinates of a generic point on the sphere, and R is its radius.

- centre at an arbitrary point in space, $P(a, b, c)$,

$$(x - a)^2 + (y - b)^2 + (z - c)^2 = R^2,$$

where (x, y, z) are the coordinates of a generic point on the sphere, and R is its radius.

3. **cylinders:** were defined as infinite straight tubes with circular cross-section. We encountered three basic types

$$x^2 + y^2 = R^2 \quad (\text{radius } R \text{ and the } z\text{-axis as its axis}),$$

$$x^2 + z^2 = R^2 \quad (\text{radius } R \text{ and the } y\text{-axis as its axis}),$$

$$y^2 + z^2 = R^2 \quad (\text{radius } R \text{ and the } x\text{-axis as its axis}),$$

where R is a given scalar.