

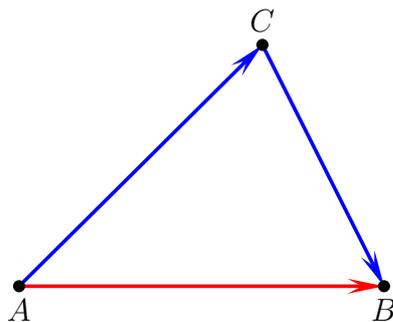
Mathematics 1T (Algebra)

Summary of Week #1

- A **vector** is an object that possesses
 1. a *magnitude* (which is always positive)
 2. a *direction* in space
- **OBS.** the only exception is the *zero vector* which has magnitude zero and no direction.
- In print, vectors are usually denoted by symbols in bold type (\mathbf{u} , \mathbf{v} , \mathbf{w} , etc) and the zero vector by $\mathbf{0}$. In handwriting, the symbols are often underlined: \underline{u} , \underline{v} , \underline{w} , etc.
- Two vectors are equal to each other if they have the *same magnitude* **and** the *same direction*.
- The **negative** of a vector \mathbf{u} , denoted by $-\mathbf{u}$, is defined to be the vector with the same magnitude as \mathbf{u} but the opposite direction.
- A vector of magnitude 1 is called a **unit** vector.
- There are two different ways in which vectors can be presented:
 1. a **geometric** way: vectors are discussed using their *representatives* (see the stuff about directed line segments in your notes).
 2. an **algebraic** way: this will be done in *Week #2*.
- Vector arithmetic: we have discussed three important arithmetic operations:
 1. *addition* of two vectors \mathbf{u} and \mathbf{v} , $\mathbf{u} + \mathbf{v}$. This operation was defined using the **Triangle Law** which can be seen at work below. For example,

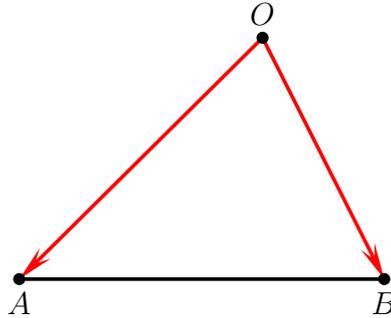
$$\vec{AB} = \vec{AC} + \vec{CB}.$$

2. *subtraction* of two vectors \mathbf{u} and \mathbf{v} , $\mathbf{u} - \mathbf{v}$. Note that $\mathbf{u} - \mathbf{v} = \mathbf{u} + (-\mathbf{v})$.
3. *multiplication* of a vector \mathbf{u} by a scalar $\lambda \in \mathbb{R}$, $\lambda\mathbf{u}$.



- The **position vector** of a point P (arbitrary) relative to the origin O (fixed) is \overrightarrow{OP} .
- An important result is that for any points A and B , we have

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$$



- Non-zero vectors are **parallel** if they have the same or opposite directions (by convention, $\mathbf{0}$ is regarded as being parallel to every vector).
- If \mathbf{u} is a non-zero vector, then the vectors parallel to \mathbf{u} are precisely the scalar multiples $\lambda\mathbf{u}$, where λ ranges through the real numbers.
- If P is a point on a line AB , then the vectors \overrightarrow{AP} and \overrightarrow{PB} are parallel, so that

$$\overrightarrow{AP} = \lambda\overrightarrow{PB}, \quad (1)$$

for some scalar λ . We call λ the **position ratio of P with respect to A, B** , and often write

$$\frac{AP}{PB} = \lambda. \quad (2)$$

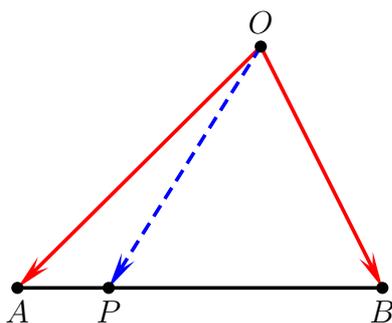
OBS. It is important to realise that (2) is just a convenient notation for writing (1) (remember that vectors cannot be divided). In (2) we are not quite talking about the ratio of the distance AP to the distance PB . We are actually talking of the ratio of the *signed* distances AP and PB , where we regard a distance as positive if it is measured in the direction from A to B , and negative if it is measured in the opposite direction.

- **Section Formula:** Let P be a point on the line AB with position ratio

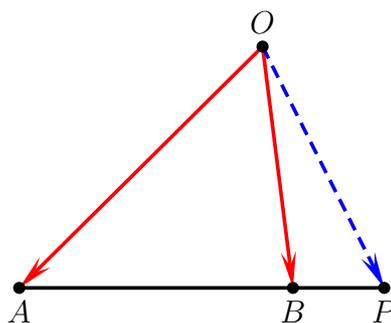
$$\frac{AP}{PB} = \frac{m}{n} \quad (\text{as defined above}).$$

Then \overrightarrow{OP} , the position vector of P , is given by

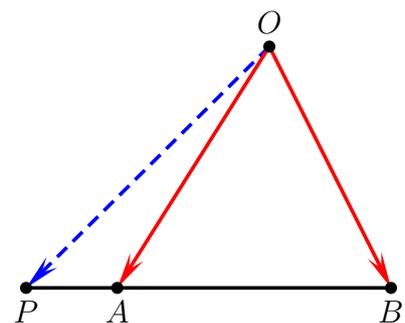
$$\overrightarrow{OP} = \frac{n\overrightarrow{OA} + m\overrightarrow{OB}}{m+n}.$$



Case (I)



Case (II)



Case (III)

- **OBS.** The Section Formula is still valid when m or n is negative. In **Case (I)** above the position ratio of P relative to A, B is positive (in this case both m and n are positive), whereas in **Case (II)** and **(III)** this position ratio is negative (one of the numbers m and n must be negative).
- **Collinearity:** Three points P, Q, R are said to be *collinear* if they lie on the same line.
- To prove that three given points P, Q, R are collinear it is sufficient to prove that

$$\overrightarrow{PQ} = (\text{some scalar}) \cdot \overrightarrow{QR}$$