ELASTICITY DAY - Wednesday 5 June 2024

University of Huddersfield, Oastler Building (OA7/29 & OA7/31)

Organizer: Ciprian D. Coman (cdc3p@yahoo.com)

09:45 – 10:00	*Welcome and refreshments*
10:00 – 10:25	Art Gower (Sheffield) **Monitoring elastic waves to predict forces in bearings**
10:25 – 10:50	Mario Lazaro (Valencia) **The Born approximation applied to wave propagation in elastic plates**
10:50 – 11:15	John Chapman (Keele) **Near-field scattering by the method of locally subsonic waves**
11:15 – 11:35	*Coffee break (20 mins.)*
11:35 – 12:00	Ian Thompson (Liverpool) **Energy flux in elastic plates**
12:00 – 12:25	Raimondo Penta (Glasgow) **Asymptotic homogenization for multi-scale composites. Theoretical issues and applications to aged bone**
12:25 – 12:50	Davood Shahsavari (Glasgow) **Surface instability of a finitely deformed magneto-elastic half-space**
12:50 – 13:40	*Lunch break (50 mins.)*
13:40 – 14:05	Michael Nieves (Keele) **Dynamic characterization of multi-scale flexural structures**
14:05 – 14:30	Richard Wiltshaw (Imperial College) **Analytical solutions for Bloch waves in resonant phononic crystals**
14:30 – 14:55	Matt Tranter (Nottingham Trent) **Delamination detection in layered waveguides via scattering of Ostrovsky wave packets**

14:55 – 15:25	*Coffee break (30 mins.)*

15:25 – 15:50	Karima Khusnutdinova (Loughborough) **Undular bores generated by fracture**
15:50 – 16:15	Sheeru Shamsi (Keele) **Asymptotic models for a fluid-loaded elastic layer**
16:15 – 16:40	Peter Wootton (Keele) **The effect of contact conditions on the performance of flexural meta-surfaces**

16:40 – 16:45	*Closing remarks*

List of participants:

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Abstracts:

Monitoring elastic waves to predict forces in bearings

Art Gower, Sheffield

When attaching an acoustic, or elastic, sensor onto the raceway of a rolling element bearings, the predominant signals received will be noise due to the pressure and shear forces between the bearings and the raceway. This can be a positive: if the aim is to measure these forces, then a passive sensing method will be sensitive to them. However, if the goal is to measure abnormal sounds, such as those emitted by a defect or crack when a bearing passes close by, then the signal from the defects can be drowned out by the noise from the bearings.

There is a number of protocols, under the banner of condition monitoring, that have developed to interpret the vibrations of rolling element bearings, and defect can be detected from them. To our surprise, it appears that no one has developed a full elastic wave mathematical method to fully describe and understand these vibrations. In this talk we illustrate this model and how it can be used to develop new and more accurate sensors.

The model we develop is broadly a tomographic method. Tomographic methods are often impractical, as they can require a large number of sensors and data. However, for cylindrical roller bearings, there is a significant prior information available: the number of bearings, the speed they travel, cylindrical symmetry, points of contact, etc. Using this prior information means that a tomographic method can be used with very few sensors, as we demonstrate with numerical simulations in this talk.

The Born approximation applied to wave propagation in elastic plates

Mario Lazaro, Valencia

In recent decades, notable advancements have occurred in the realm of phononic crystals, thanks to innovative methods for manipulating waves in intricate mediums. Of particular interest is the domain of materials consisting of a host medium interspersed with an array of point scatterers. In particular, this presentation focuses on wave propagation behaviour in elastic plates. Procedures based on the multiple scattering equations allow the analytical solution of the problem in the most general case. We are interested in determining the mathematical conditions that allow us to differentiate when an array of scatterers produces weak scattering on a plate. To accomplish this objective, we propose determining the spectral radius of the scattering matrix, whose value closely correlates with the classical Born approximation, as applied herein to elastic waves.

Near-field scattering by the method of locally subsonic waves

John Chapman, Keele

A technique will be developed for determining the wave field scattered by a compact body when it is close enough to a source to be in its near field. The approach is based on the fact that large regions of many near fields may be well approximated at each point in space by a subsonic plane wave (also called an inhomogeneous plane wave, or an evanescent wave). Such a wave is defined by the property that in one direction it propagates with subsonic phase speed, while in a perpendicular direction it has exponential amplitude variation. Hence by defining a canonical problem, compact scattering of a subsonic plane wave, and solving it, a unified analytical treatment becomes available for many near-field scattering problems.

The approach draws on the formulae of Rayleigh scattering (as applied to an incident field with complex wavenumber) and the asymptotic theory of the wave equation. For an arbitrary three-dimensional multipole, it will be shown in full detail how its subsonic wave structure depends on the spherical harmonic parameters (m, n), and that the results have a large region of validity. The method applies equally to elastic, acoustic, and electromagnetic waves.

The work is collaborative with **S. C. Hawkins** ***(****MacQuarie University),* and was begun at an Isaac Newton Institute programme on wave scattering at Cambridge University. It is under consideration for publication in the Proceedings of the Royal Society (A).

Asymptotic homogenization for multi-scale composites; theoretical issues and applications to aged bone.
Raimondo Penta, Glasgow

We discuss recent theoretical and computational advances [1, 2] in the context of multiscale composites, and their application to the bone hierarchical structure. Recent experimental data revealed a stiffening of aged cortical bone tissue, which could not be explained by common multiscale elastic material models. We explain these data by incorporating the role of mineral fusion via a new hierarchical modeling approach exploiting the asymptotic (periodic) homogenization (AH) technique for three-dimensional linear elastic composites. We quantify for the first time the stiffening that is obtained by considering a fused mineral structure in a softer matrix in comparison with a composite having non-fused cubic mineral inclusions. We integrate the AH approach in the Eshelby-based hierarchical mineralized turkey leg tendon model [3], which can be considered as a base for musculoskeletal mineralized tissue modeling. We model the nest scale compartments, i.e. the extra-fibrillar space and the mineralized collagen fibril, by replacing the self-consistent scheme with our AH approach. This way, we perform a parametric analysis at increasing mineral volume fraction, by varying the amount of mineral that is fusing in the axial and transverse tissue directions in both compartments. Our effective stiffness results are in good agreement with those reported for aged human radius and support the argument that the axial stiffening in aged bone tissue is caused by the formation of a continuous mineral foam. Moreover, the proposed theoretical and computational approach supports the design of biomimetic materials which require an overall composite stiffening without increasing the amount of the reinforcing material [4].

References:

[1] Penta, R. and Gerisch, A. (2017). The asymptotic homogenization elasticity tensor properties for composites with material discontinuities. Continuum Mechanics and Thermodynamics, 29(1), 187-206.
[2] Penta, R. and Gerisch, A. (2015). Investigation of the potential of asymptotic homogenization for elastic composites via a three-dimensional computational study. Computing and Visualization in Science, 17(4), 185-201.
[3] Tiburtius, S., Schrof, S., Molnar, F., Varga, P., Peyrin, F., Grimal, Q., and Gerisch, A. (2014). On the elastic properties of mineralized turkey leg tendon tissue: multiscale model and experiment. Biomechanics and modeling in mechanobiology, 13(5), 1003-1023.
[4] Penta, R., Raum, K., Grimal, Q., Schrof, S., and Gerisch, A. (2016). Can a continuous mineral foam explain the stiffening of aged bone tissue? A micromechanical approach to mineral fusion in musculoskeletal tissues. Bioinspiration and biomimetics, 11(3), 035004

Surface instability of a finitely deformed magneto-elastic half-space

Davood Shahsavari, Glasgow

We develop a mathematical model to analyze the surface instabilities of an incompressible magnetoelastic half-space. Based on a consistent variational formulation that accounts for magnetic field inside and outside the material, we derive the instability criteria for developing surface wrinkles. We present new results for a Mooney-Rivlin type constitutive model and show that instability regime can be tuned by applied magnetic field and constitutive parameters.

Dynamic characterization of multi-scale flexural structures

Michael Nieves, Keele

The dynamic behavior of asymmetric flexural systems, involving a master beam attached to a non-periodic collection of flexural resonators, is discussed [1]. The resonators couple longitudinal and flexural responses of the master beam. Its response is described via Green's functions, with intensities determined from an algebraic system embedding interactions of individual resonators. For infinite periodic waveguides, we derive an effective model called the generalized Rayleigh beam that supports flexural-longitudinal wave coupling. This effective model, derived through the meso-scale method of asymptotic analysis [2], is efficient in regimes not typically encountered in existing homogenization approaches [3].

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References:
[1] Nieves MJ and Movchan AB. 2022. Meso-scale method of asymptotic analysis of elastic vibrations in periodic and non-periodic multi-structures. Quarterly Journal of Mechanics and
Applied Mathematics 75 (3), 171-214.
[2] Mazya, V, Movchan, AB, Nieves, MJ, 2013: Green's Kernels and Meso-scale Approximations in Perforated Domains, Lecture Notes in Math. 2077, Springer
[3] Nieves M and Movchan A. 2023. Asymptotic Theory of Generalized Rayleigh Beams and the Dynamic Coupling. In: Altenbach, H., Prikazchikov, D., Nobili, A. (eds) Mechanics of High-Contrast Elastic Solids., Advanced Structured Materials, vol 187. Springer, Cham.

Analytical solutions for Bloch waves in resonant phononic crystals

Richard Wiltshaw, London

I will discuss the canonical problem of wave scattering by arrays of Neumann inclusions. Firstly, we consider the wavefield to be governed by the Helmholtz equation. We apply the method of matched asymptotic expansions to show how small scatterers can be modelled as singular perturbations to the free space. Analytical expressions then follow in terms of singular Green's functions, from which we construct an eigenvalue problem to consider Floquet-Bloch waves, or we can consider scattering problems as an extension to Foldy's method. The methods presented allow for efficient, rapid and accurate computations.

These methods will then be applied in an elastic setting to consider waves propagating through an elastic plate, whose surface is patterned by periodic arrays of elastic beams. Our methodology is versatile and allows us to solve a range of problems regarding arrangements of multiple beams per primitive cell, over Bragg to deep-subwavelength scales. We cross-verify against finite element numerical simulations to gain further confidence in our approach. The accuracy and flexibility of our solutions are demonstrated by engineering topologically non-trivial states, from primitive cells with broken spatial symmetries, following the phononic analogue of the Quantum Valley Hall Effect. These topologically non-trivial states exist near flexural resonances of the constituent beams of the phononic crystal and hence can be tuned into a deep-subwavelength regime.

Delamination detection in layered waveguides via scattering of Ostrovsky wave packets

Matt Tranter, Nottingham

For layered waveguides with bonding between the layers, it is imperative that the bonding remains intact, otherwise they could suffer catastrophic failure under stress. Detecting faults in the bonding layer, such as delamination, is of paramount importance. In this talk we will explore recent developments in the detection of delamination in a two-layered waveguide with a soft bonding between the layers. We assume that the delaminated section of the waveguide is sandwiched between two bonded sections. Using a layered lattice model, the strains in the waveguide can be described by coupled regularized Boussinesq equations. A semi-analytical approach consisting of matched asymptotic multiple-scale expansions is used and, assuming the layers have distinct properties, leads to Ostrovsky equations in soft bonded regions and Korteweg-de Vries equations in the delaminated region. Ostrovsky wave packets are generated from an incident strain solitary wave and these are shown to be in good agreement with direct numerical simulations. Analysis of the phase shift in the Ostrovsky wave packet, introduced from the delaminated region, allows us to predict both the position and length of the delamination, a novel development in the field. These results motivate experiments to validate the theoretical results, with the aim of creating a tool to monitor the integrity of layered structures.

Undular bores generated by fracture

Karima Khusnutdinova, Loughborough

Undular bores, or dispersive shock waves, are non-stationary waves propagating as oscillatory transitions between two basic states, in which the oscillatory structure gradually expands and grows in amplitude with distance travelled. The generation of undular bores in polymethylmethacrylate (PMMA) bars following tensile fracture is observed using high-speed pointwise photoelasticity. We show that a viscoelastic extended Korteweg-de Vries (veKdV) equation provides good agreement with the key observed experimental features of the generated bores for a suitable choice of material parameters. Linearization of the veKdV equation near the pre-strain level prior to fracture captures some features at the front of the bore. We analyze the behavior of

the bores following both natural and induced tensile fracture. We also vary the width of the waveguide and conditions at fracture. Such waves could be present in the signals generated by fracking, earthquakes and other events involving transverse fracture of an appropriately pre-strained waveguide. Joint work with **Curtis Hooper**, **Pablo Ruiz** and **Jonathan Huntley**.

Asymptotic models for a fluid-loaded elastic layer

Sheeru Shamsi, Keele

The classical problem for an elastic layer immersed into a compressible fluid is revisited starting from the general asymptotic perspective. The results of the low-frequency analysis are discussed. The adapted scaling corresponds to the so-called fluid-borne bending wave. Approximate equations are derived at various orders, up to the third one. The first order approximation corresponds to the traditional formulation for a thin Kirchhoff plate submerged into an incompressible fluid. It is worth noting that the plate inertia can be neglected at leading (zero) order. Various higher-order corrections to the aforementioned setup based on Kirchhoff theory appear at second and third orders. Specifically, the transverse shear deformation has to be taken into consideration at second order along with an appropriate asymptotic correction in the impenetrability condition, whereas the plate rotatory inertia and the fluid compressibility have to be incorporated only at the third order. Finally, the associated approximate dispersion relations are compared with the "exact" dispersion relation.

Energy flux in elastic plates

Ian Thompson, Liverpool

Any valid formula for energy flux must satisfy two necessary conditions:
(1) The flux across a closed contour (surface in 3D) containing no sources must evaluate to zero.
(2) The flux across a boundary where no transmission occurs must also evaluate to zero.
An energy flux formula for time-harmonic waves in a thin elastic plate modelled by classical (Kirchhoff) theory was derived in [1], but shortly afterwards it was pointed out that it has terms missing [2]. In fact, it may fail to satisfy condition (1), though only if the contour is not smooth. In their response [3], the authors of [1] noted that the corrections in [2] produce a formula that does not give zero flux across a free edge; it always satisfies condition (1), but fails to satisfy condition (2).

In this presentation we will show how the discrepancy above may be understood using Mindlin plate theory. Results for Kirchhoff theory (satisfying both (1) and (2)) are obtained from the leading-order asymptotics as the wavenumber tends to zero.

References:

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[2] Calculation of the power flow in flexural waves on thin plates. Y. I. Bobrovnitskii, Journal of Sound and Vibration 194(1) 103-106, 1996. doi:10.1006/jsvi.1996.0347
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The effect of contact conditions on the performance of flexural meta-surfaces

Peter Wootton, Keele

Plane-strain motion of a flexural seismic meta-surface in the form of a regular array of thin Kirchhoff plates attached to the surface of an elastic half-space is analyzed. Two types of contact conditions, including simply supported plates and plates moving along horizontal rails are studied. Dispersion of time harmonic waves is investigated both asymptotically and numerically. It is shown that frequency band gaps are not the feature of the array composed of simply supported plates and that the scaling laws, expressed through geometric and material problem parameters, drastically differ from each other for two considered setups.