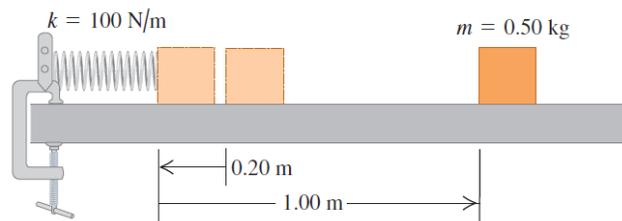


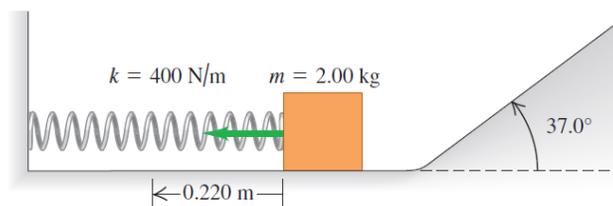
## Problem Set # 9

## Work, energy & power

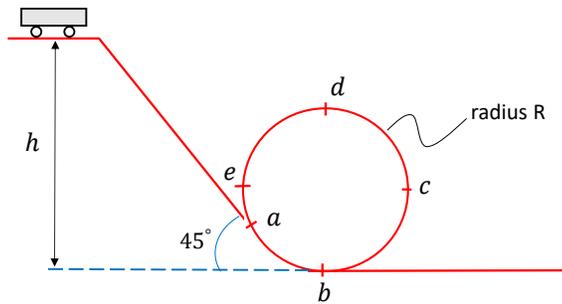
1. A block with mass 0.5 Kg is forced against a horizontal spring of negligible mass, compressing the spring a distance of 0.2 m, as seen in the sketch included below. When released, the block moves on a horizontal tabletop for 1.0 m before coming to rest. The spring constant  $k$  is 100 N/m. What is the coefficient of kinetic friction  $\mu_k$  between the block and the tabletop?



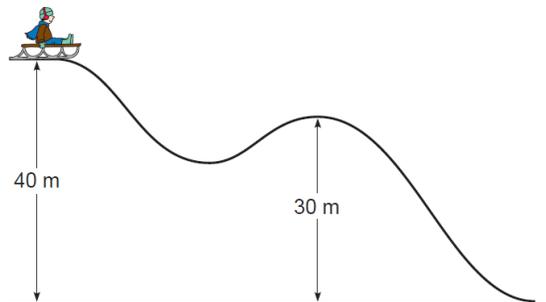
2. A 2.0 Kg block is pushed against a spring with negligible mass and force constant  $k = 400$  N/m, compressing it 0.220 m. When the block is released, it moves along a frictionless, horizontal surface and then up a frictionless incline with slope  $37^\circ$ .
- (a) What is the speed of the block as it slides along the horizontal surface after having left the spring?
- (b) How far does the block travel up the incline before starting to slide back down?



3. A frictionless roller coaster car is released from rest at a height  $h$  and rolls down of its own accord down a  $45^\circ$  slope through the loop  $a, b, c, d, e$ . The loop is of constant radius  $R$  and has the slope as its tangent. The car then continues on a level track, also at a tangent to the loop. Answer the following questions.
- (a) Where is the magnitude of the force between the car and the track greatest? Derive an expression for the force at this point.
- (b) Where is the force the smallest? Derive an expression for the force at this point.



4. A sled and rider with total mass of 40 Kg are perched at the top of a hill as pictured in the diagram. The top of this hill is 40 m above the low point in the path of the sled. A second hump in the hill is 30 m above the low point. Suppose that we also know that approximately 2000 J of work is done against friction as the sled travels between these two points.
- Will the sled make it to the top of the second hump if no kinetic energy is given to the sled at the start of its motion? Explain.
  - What is the maximum height that the second hump could be in order for the sled to reach the top, assuming that the same work of friction will be involved and that no initial push is provided? Explain.

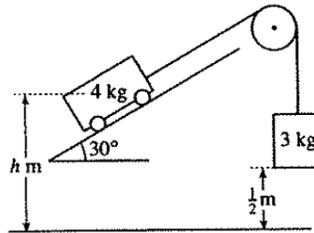


5. A particle of mass  $m$  moves in a straight line under the action of a constant force. Show that the time average of the kinetic energy of the particle in any interval of time is

$$\frac{1}{6}m(v_1^2 + v_1v_2 + v_2^2),$$

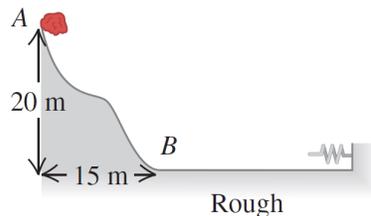
where  $v_1$  and  $v_2$  are the initial and final speeds in the aforementioned time interval.

6. A toy has the form of a truck of mass 4.0 Kg which can run on a track at an angle of  $30^\circ$  to the horizontal. A light chain attached to the truck runs parallel to the track, passes over a light pulley at its upper end, and then hangs vertically (as seen in the diagram). A counterweight of mass 3.0 Kg is attached to the free end of the chain. The system is released from rest with the counterweight at a height 0.5 m above the floor. Find how fast the truck is moving when the counterweight hits the floor.

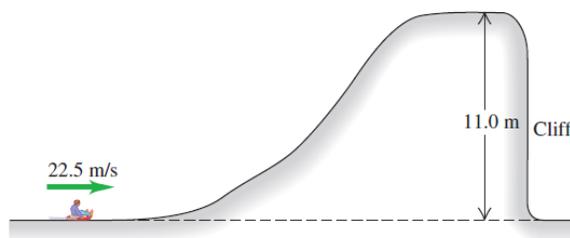


7. A 15.0 Kg stone slides down a snow-covered hill – see diagram, leaving point  $A$  with a speed of 10 m/s. There is no friction on the hill between points  $A$  and  $B$ , but there is friction on the level ground at the bottom of the hill, between  $B$  and the wall. After entering the rough horizontal region, the stone travels 100 m and then runs into a very long, light spring with force constant 2.0 N/m. The coefficients of kinetic and static friction between the stone and the horizontal ground are 0.2 and 0.8, respectively.

- What is the speed of the stone when it reaches point  $B$ ?
- How far will the stone compress the spring?
- Will the stone move again after it has been stopped by the spring?



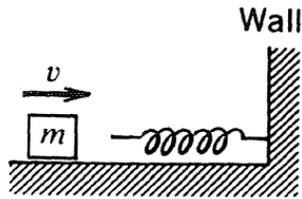
8. A sled with rider having a combined mass of 125 Kg travels over the perfectly smooth icy hill shown below. How far does the sled land from the foot of the cliff?



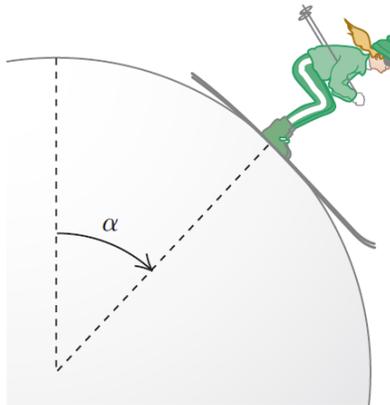
9. A non-ideal elastic spring exerts a restoring force given by

$$F(x) = -k_1x - k_2x^3, \quad (k_1, k_2 > 0),$$

where  $x$  is the deviation from its unstretched length. The spring rests on a frictionless surface, and a frictionless block of mass  $m$  and initial speed  $v$  hits a spike on the end of the spring and sticks (see sketch). How far does the mass travel, after being impaled, before it comes to rest? (Assume the mass of the spring negligible.)

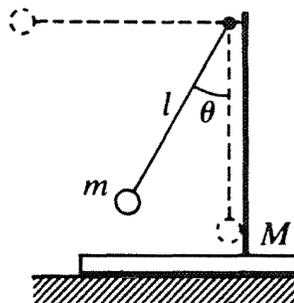


10. A skier starts at the top of a very large, frictionless snowball, with a very small initial speed, and skis straight down the side (as seen in the diagram). At what point does she lose contact with the snowball and fly off at a tangent? That is, at the instant she loses contact with the snowball, what angle  $\alpha$  does a radial line from the centre of the snowball to the skier make with the vertical?



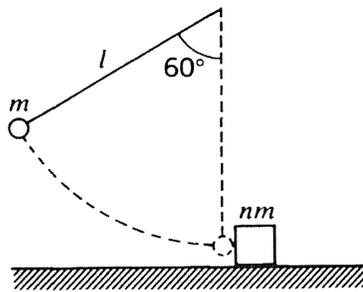
11. A small ball of putty, of mass  $m$ , is attached to a string of length  $\ell$  fastened to an upright on a wooden board resting on a horizontal table (see diagram). The combined mass of the board and upright is  $M$ . The friction coefficient between the board and the table is  $\mu$ ; assume that static and kinetic friction coefficients are the same. The ball is released from rest with the string in a horizontal position. It hits the upright in a completely inelastic collision. While the ball swings down, the board does not move.

- (a) How far does the board move after the collision?
- (b) What is the minimum value that  $\mu$  must have to prevent the board from moving to the left while the ball swings down? Assume  $m \ll M$  and give the critical condition at  $\theta = 45^\circ$ .



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12. A pendulum bob of mass  $m$ , at the end of a string of length  $\ell$ , starts from rest at the position shown in the diagram, with the string at  $60^\circ$  to the vertical. At the lowest point of the arc the bob strikes a previously stationary block, of mass  $nm$ , that is on a frictionless horizontal surface. The collision is perfectly elastic.

- (a) What is the speed of the bob just before the impact occurs?  
(b) What is the tension in the string at this instant?  
(c) What speed is given to the block by the impact?  
(d) In the oscillations of the pendulum after the collision, what maximum angle  $\theta$  to the vertical does the string make? (Obtain your answer in the form  $\cos \theta = \dots$  )



13. A non-ideal elastic spring exerts a restoring force

$$F(x) = -k_1x - k_2x^2$$

if it is stretched or compressed, where  $k_1 = 60.0 \text{ N/m}$  and  $k_2 = 18.0 \text{ N/m}^2$ . The mass of the spring is negligible.

- (a) Calculate the potential energy  $U(x)$  for this spring; let  $U = 0$  when  $x = 0$ .  
(b) An object with mass  $0.9 \text{ Kg}$  on a frictionless, horizontal surface is attached to this spring, pulled a distance  $1.0 \text{ m}$  to the right (the  $+x$ -direction) to stretch the spring, and then released. What is the speed of the object when it is  $0.5 \text{ m}$  to the right of the  $x = 0$  equilibrium position (i.e., the position in which the length of the spring is relaxed).