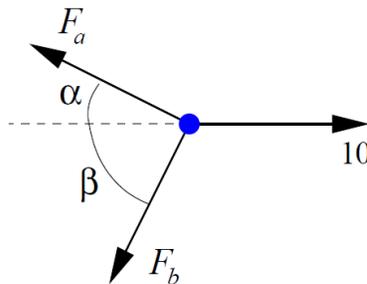


## Problem Set # 3

## Statics

1. An object is subject to three forces, as shown in the diagram; one force has magnitude 10, while the other forces are of magnitudes  $F_a$  and  $F_b$ . The three forces act in the same plane, and the object is in equilibrium. The angle  $\alpha$  is known and is equal to  $30^\circ$ ; the angle  $\beta$  is at our disposal.



- (a) By introducing the usual unit vectors  $i$  and  $j$  respectively parallel and perpendicular to the force of magnitude 10, and considering components in these two directions, show that the conditions for equilibrium can be written in the form

$$10 = \frac{1}{2}\sqrt{3}F_a + F_b \cos \beta, \quad \frac{1}{2}F_a = F_b \sin \beta,$$

and hence find  $F_b$  as a function of the angle  $\beta$ .

- (b) What value of  $\beta$  minimises  $F_b$ ? Find the corresponding minimum value.
2. An object is in equilibrium subject to three applied forces. The first force is  $\mathbf{F}_1 = 4\mathbf{i} + 3\mathbf{j}$ . The other two forces  $\mathbf{F}_2$ ,  $\mathbf{F}_3$  are parallel to the vectors

$$\hat{\mathbf{a}} = \frac{1}{3}(-2, -2, 1) \quad \text{and} \quad \hat{\mathbf{b}} = \frac{1}{\sqrt{6}}(-2, -1, -1).$$

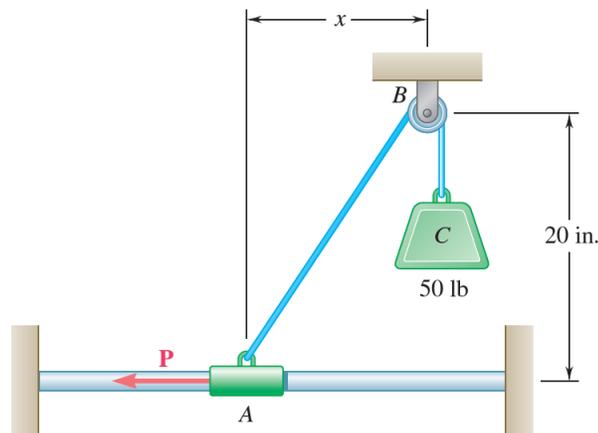
- (a) Check that  $\hat{\mathbf{a}}$  and  $\hat{\mathbf{b}}$  are each of unit length.
- (b) By writing down the (vector) force balance equation for the object to be in equilibrium, determine  $|\mathbf{F}_2|$  and  $|\mathbf{F}_3|$ .
3. Four forces act on an object, which is in equilibrium. The forces act in the directions of the following unit vectors:

$$\hat{\mathbf{a}} = (0, 0, 1), \quad \hat{\mathbf{b}} = \frac{1}{5}(4, 3, 0), \quad \hat{\mathbf{c}} = \frac{1}{3}(-2, -2, 1), \quad \hat{\mathbf{d}} = \frac{1}{5}(-3, 4, 0).$$

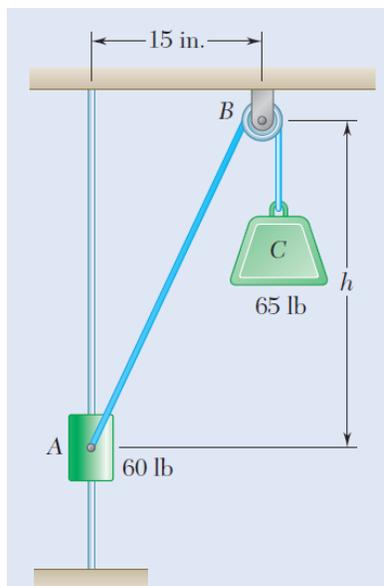
Check that each unit vectors is indeed of unit length. If the force parallel to  $\hat{\mathbf{a}}$  has magnitude 10 N, what are the magnitudes of the other forces?

[**Hint:** Write down the force balance equation for equilibrium and consider each component; it will help if you assign symbols to the unknown magnitudes (e.g.,  $B$ ,  $C$ , and  $D$ ). ]

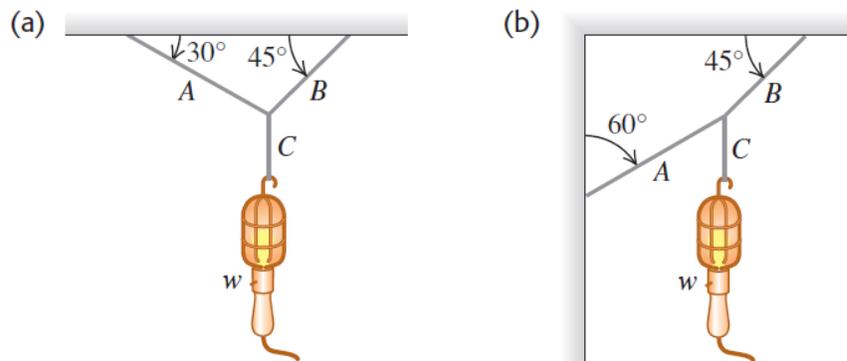
4. Collar  $A$  is connected as shown to a 50-lb load and can slide on a frictionless horizontal rod. Determine the magnitude of the force  $P$  required to maintain the equilibrium of the collar when: (a)  $x = 4.5$  in.; (b)  $x = 15$  in.



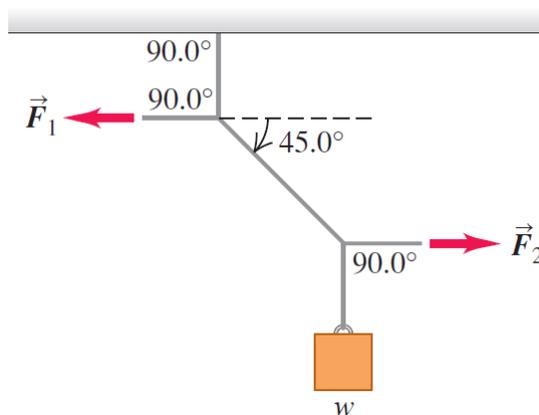
5. The 60-lb collar  $A$  can slide on a frictionless vertical rod and is connected as shown below to a 65-lb counterweight  $C$ . Determine the value of  $h$  for which the system is in equilibrium.



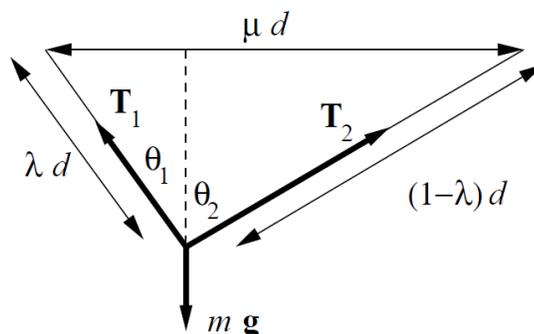
6. Find the tension in each cord in the sketch below, if the weight of the suspended object is  $w$ .



7. In the sketch below the tension in the diagonal string is 30 N.



- (a) Find the magnitudes of the horizontal forces  $F_1$  and  $F_2$  that must be applied to hold the system in equilibrium.
- (b) What is the weight  $w$  of the suspended block?
8. A string of length  $d$  hangs between two horizontal points a distance  $\mu d$  apart, where  $0 < \mu < 1$ . A particle of mass  $m$  is attached to the string a distance  $\lambda d$  from one end, where  $0 < \lambda < 1$ , such that the two parts of the string make angles  $\theta_1$  and  $\theta_2$  to the vertical. The particle's weight is balanced by the tensions  $T_1$  and  $T_2$  in the string.

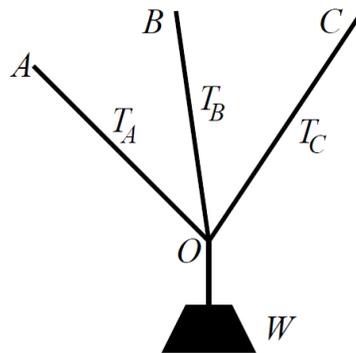


- Find  $T_1$  and  $T_2$  in terms of  $m$ ,  $\theta_1$ , and  $\theta_2$ .
- When  $\lambda = 1/2$ , show that

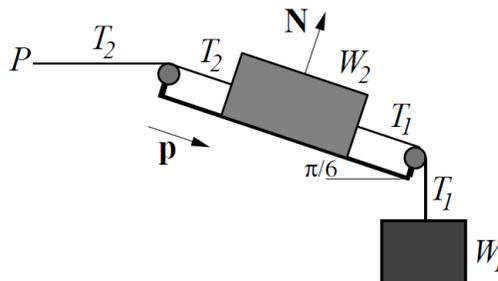
$$T_1 = T_2 = \frac{1}{2}mg(1 - \mu^2)^{-1/2}.$$

9. A heavy object of weight  $W$  is suspended from the point  $O$ , which is tied to the ceiling by three ropes:  $OA$ ,  $OB$ ,  $OC$ . The system is in equilibrium. If we take the point  $O$  as the origin, then the coordinates of the tethering points on the ceiling are

$$A(2, 3, 6), \quad B(-2, -9, 6), \quad C(-3, 6, 6).$$

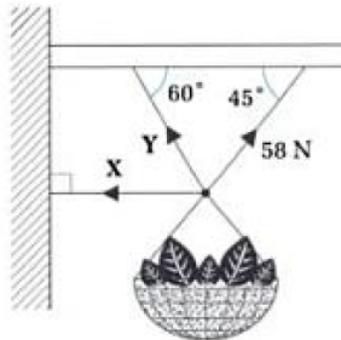


- (a) Write down the vectors  $\vec{OA}$ ,  $\vec{OB}$  and  $\vec{OC}$ .
- (b) Find the unit vectors  $\hat{a}$ ,  $\hat{b}$  and  $\hat{c}$  in the directions of  $\vec{OA}$ ,  $\vec{OB}$  and  $\vec{OC}$ , respectively.
- (c) The tensions in the ropes are denoted by  $T_A$ ,  $T_B$ ,  $T_C$  (see the sketch below). Explain why the tension forces acting on the object can be written in the form  $T_A\hat{a}$ ,  $T_B\hat{b}$ ,  $T_C\hat{c}$ .
- (d) By considering the (vector) force balance equation for the system described above, find  $T_A$ ,  $T_B$  and  $T_C$  in terms of the weight  $W$ . Which rope has the greatest tension?
10. Two heavy blocks are held in equilibrium by the arrangement shown in the diagram below. The rope is held in place by a horizontal force of magnitude  $T_2$  applied at the point  $P$ . The incline is *smooth* (i.e., there is no friction between the block and the incline) and the pulleys are *frictionless* (i.e., the tensions in the rope are the same on either side of the pulleys). The angle between the incline and the horizontal is  $\pi/6$ .

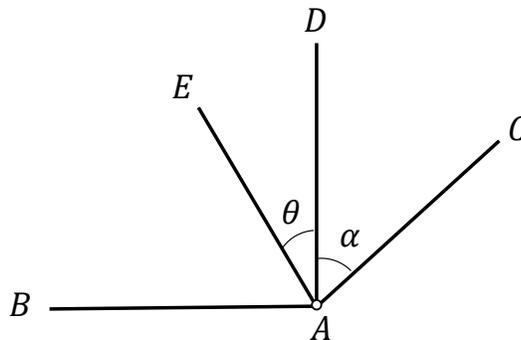


- (a) Explain why  $T_1 = W_1$ .
- (b) List the four forces acting on the block with weight  $W_2$ .
- (c) Write down the (vector) force balance equation for the block  $W_2$ . By resolving this vector equation in the direction parallel to the incline (defined by the unit vector  $\mathbf{p}$ ) find the force  $T_2$  required to keep both blocks in place.

11. A hanging basket has been suspended from a boom outside a shop. The mass of the hanging basket is 7.5 Kg. Find the magnitude of the forces  $\mathbf{X}$  and  $\mathbf{Y}$ .

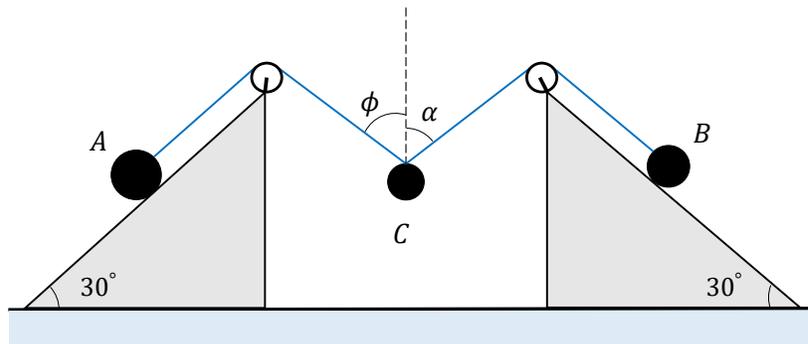


12. A particle of weight 75 N is hanging in equilibrium at a point  $A$  supported by four strings  $AB$ ,  $AC$ ,  $AD$ ,  $AE$ , all in the same vertical plane (see sketch below). The string  $AB$  is horizontal and the tension in  $AB$  is 13 N; the string  $AC$  makes an angle  $\alpha$  with the vertical, where  $\sin \alpha = 12/13$ , and the tension in  $AC$  is 26 N; the string  $AD$  is vertical and the tension in  $AD$  is 5 N; the string  $AE$  makes an angle  $\theta$  with the vertical. Find the tension in this string, as well as the angle  $\theta$ .



[**Hint:** Draw your own diagram and indicate all the forces that act on the system considered in this problem; write down the (vector) force balance equation, etc. ]

13. The diagram shows masses  $A$  and  $B$  each lying on smooth planes of inclination  $30^\circ$ .

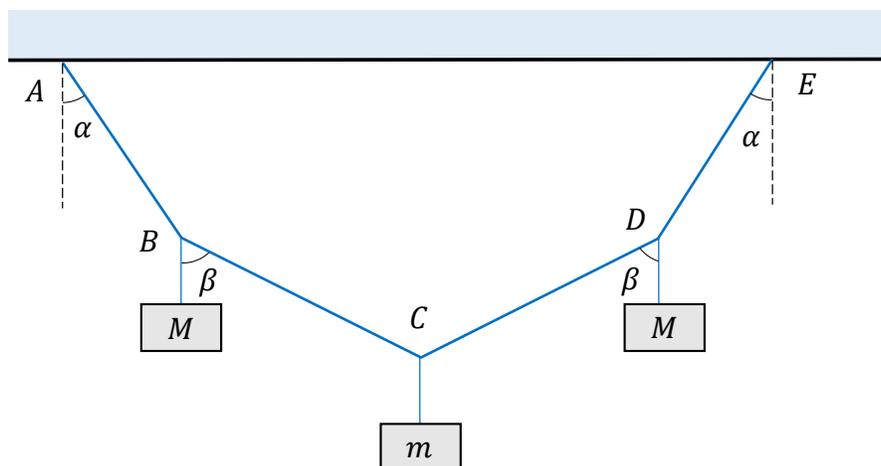


Light inextensible strings attached to  $A$  and  $B$  pass along the lines of greatest slope, over smooth pulleys and are connected to a third mass  $C$  hanging freely. The strings make angles  $\phi$  and  $\alpha$  with the upward vertical as shown. If  $A$ ,  $B$  and  $C$  have masses  $2m$ ,  $m$  and  $m$ , respectively, and the system rests in equilibrium, show that

$$\sin \alpha = 2 \sin \phi, \quad \cos \alpha + 2 \cos \phi = 2.$$

Hence find  $\phi$  and  $\alpha$ .

14. The diagram represents a light inextensible string  $ABCDE$  in which  $AB = BC = CD = DE$ , and to which are attached masses  $M$  and  $m$  at the points  $B$ ,  $C$ , and  $D$ , respectively. The system hangs freely in equilibrium with the ends  $A$  and  $E$  of the string fixed in the same horizontal line.  $AB$  and  $DE$  each make an acute angle  $\alpha$  with the vertical such that  $\tan \alpha = 3/4$ .  $BC$  and  $CD$  each make an acute angle  $\beta$  with the vertical such that  $\tan \beta = 12/5$ .



- (a) By considering the forces acting at  $C$ , calculate the tension in  $BC$  in terms of  $m$  and  $g$ .
- (b) By considering the forces acting at  $B$ , calculate the tension in  $AB$  in terms of  $M$  and  $g$ . Find also the ratio of the two masses  $M/m$ .