Subject Area: Analysis, Differential Equations

Title: Matched Asymptotic Expansions

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Description:

This project deals with the following common situation encountered in many branches of science. Suppose that we need to solve a certain problem $\mathcal{P}(u,\, x\,;\varepsilon)$, which amounts to finding the expression of a function $u \equiv u(x;\,\varepsilon)$ defined on $\Omega \subset \mathbb{R}$, as $\varepsilon \to 0$. For example, the problem can be a differential equation or it may involve evaluating a complicated integral. Due to the presence of the (very) small parameter $\varepsilon > 0$, more often than not it is possible to simplify the original problem $\mathcal{P}$ in several ways, and obtain closed-form partial solutions valid on certain sub-domains of Ω which depend on ε.

To be definite, let us assume that $\mathcal{P}$ can be broken down into two simpler problems, say, $\mathcal{P}_I(u_1,\, x\,;\varepsilon)$ valid on Ω_I^ε, and $\mathcal{P}_O(u_2,\, x\,;\varepsilon)$ valid on Ω_O^ε, where $\Omega_I^\varepsilon \cap \Omega_O^\varepsilon \neq \emptyset$. The solutions of these simpler problems, however, will depend on some arbitrary constants and thus these solutions are not useful unless the constants appearing in their expressions can be related to each other. The *Method of Matched Asymptotics* (MMA) is a general theory that provides a set of tools for obtaining the simplified problems, and then fixing the arbitrary constants that appear in the corresponding solutions. The originator of this method was Ludwig Prandtl who in 1904 employed it to provide an explanation of how the small viscosity of fluids can have a tremendous effect on their flow. Apparently, the method is even older, and the first to have ever used it seems to be Laplace in 1805.

This project has a theoretical flavour and represents an introduction to the concepts and ideas of MMA. Interested students are strongly encouraged to contact me before committing themselves to this work.

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References

1. M. H. Holmes *Introduction to Perturbation Methods*, Springer Verlag, 1995
2. J. G. Simmonds and J. E. Mann *A First Look at Perturbation Theory*, Dover Inc., 1998.
3. A. Georgescu *Asymptotic Treatment of Differential Equation*, Chapman & Hall, 1995.
4. M. Van Dyke *Perturbation Methods in Fluid Mechanics*, The Parabolic Press, 1975.