Subject Area: Ordinary Differential Equations

Title: WKB Approximations

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Description:

Many problems in mathematics, mechanics, physics and other natural sciences reduce to differential equations of the form

$$\varepsilon\mathbf{u}' = \mathbf{A}(x,\, \lambda)\mathbf{u}, \qquad \alpha < x < \beta, \tag{1}$$

where $\epsilon > 0$ is a small parameter. In (1), the dash denotes differentiation with respect to x, α and $\beta \in \mathbb{R}$, $\mathbf{u} = \mathbf{u}(x) \in \mathbb{R}^n$ is an n–dimensional column vector containing the unknowns, $\lambda \in \mathbb{R}$ represents some parameter, and $\mathbf{A}(x,\, \lambda;\, \varepsilon)$ is an $n \times n$ matrix whose elements are differentiable functions. It is extremely rare for a system of differential equations with variable coefficients to be integrable by quadratures. However, the presence of the small parameter ε in (1) allows this problem to be *approximately* integrable by quadratures. The methods used to achieve this are loosely known as WKB methods (named after the initials of Wentzel, Kramers, and Brillouin, three physicists who used these approximations in the 1920's, although the first to use a WKB method was the Italian astronomer F. Carlini who published a paper on planetary orbits in 1817).

The main objective of the project is to familiarise you with a range of techniques and topics that fall within the framework described by (1), and to explore some simple practical applications. There are no special requirements for this project, but a good understanding of the theory of ordinary differential equations is essential.

References

1. M. H. Holmes *Introduction to Perturbation Methods*, Springer Verlag, 1995.
2. J. G. Simmonds and J. E. Mann *A First Look at Perturbation Theory*, Dover Inc., 1998.
3. M. V. Fedoryuk *Asymptotic Analysis*, Springer Verlag, 1996.