The University of Nottingham

SCHOOL OF MATHEMATICAL SCIENCES
DEPARTMENT OF MECHANICAL, MATERIALS AND MANUFACTURING ENGINEERING

A LEVEL 1 MODULE, SPRING SEMESTER 2018-2019

MATHEMATICS FOR ENGINEERS

Time allowed TWO Hours

Candidates may complete the front cover of their answer book and sign their desk card but must NOT write anything else until the start of the examination period is announced.

Answer ALL Questions in Sections A and B.
Sections A and B are equally weighted.

Each Multiple Choice Question has a weighting of 4 marks for each correct answer. Incorrect answers and "Abstain" responses are not awarded any marks.

Only silent, self-contained calculators with a Single-Line Display or Dual-Line Display are permitted in this examination.

Dictionaries are not allowed with one exception. Those whose first language is not English may use a standard translation dictionary to translate between that language and English provided that neither language is the subject of this examination. Subject specific translation dictionaries are not permitted.

No electronic devices capable of storing and retrieving text, including electronic dictionaries, may be used.

DO NOT turn examination paper over until instructed to do so

ADDITIONAL MATERIAL: Formula Sheet, Multiple-Choice Answer Sheet, Instructions for multiple-choice questions.

INFORMATION FOR INVIGILATORS: Please collect the multiple-choice answer sheets and scripts separately at the end of the exam.

SECTION A

1. The complex number $\dfrac{1+qi}{q-i}$ where $q \in \mathbb{R}$, can be simplified to:

   (a) i,

   (b) 1,

   (c) $\dfrac{q+q^2+(q^2-1)i}{q^2+1}$,

   (d) $\dfrac{2q+(q^2+1)i}{q^2+1}$,

   (e) None of the above.

2. The limit $\lim\limits_{x\to 0} \dfrac{\sin(3x)}{\ln(1+x)}$ has the value:

   (a) Undefined,

   (b) 0,

   (c) 6,

   (d) 3,

   (e) None of the above.

3. With C_* being constants of integration, the indefinite integral $\int \sin(2\theta)\mathrm{d}\theta$ can be evaluated as:

   (a) $C_1 + \dfrac{1}{2}\sin(2\theta)$,

   (b) $C_2 + \dfrac{1}{2}\cos(2\theta)$,

   (c) $C_3 + \cos^2\theta$,

   (d) $C_4 + \sin^2\theta$,

   (e) None of the above.

4. The Taylor series of the function $f(x) = e^{-x}\ln(x-1)$ around the point $x = 2$ starts with the terms:

   (a) $f(x) = (x-2)e^{-2} - e^{-2}(x-2)^2$,

   (b) $f(x) = (x-2)e^{-2} - \frac{3}{2}e^{-2}(x-2)^2$,

   (c) $f(x) = (x-2)e^{-2} - \frac{1}{2}e^{-2}(x-2)^2$,

   (d) $f(x) = (x-2)e^{-2} - 3e^{-2}(x-2)^2$,

   (e) None of the above.

5. Given that $\lambda_1 = 1$ is an eigenvalue of the matrix $\begin{pmatrix} 3 & -4 & 0 \\ 2 & -1 & 2 \\ -2 & 4 & 1 \end{pmatrix}$ the other eigenvalues are:

   (a) $\lambda_2 = 3, \quad \lambda_3 = -3$,

   (b) $\lambda_2 = -1, \quad \lambda_3 = -3$,

   (c) $\lambda_2 = 1, \quad \lambda_3 = 3$,

   (d) $\lambda_2 = 1, \quad \lambda_3 = -3$,

   (e) None of the above.

6. The eigenvector corresponding to the eigenvalue $\lambda = 1$ of $\begin{pmatrix} 3 & -4 & 0 \\ 2 & -1 & 2 \\ -2 & 4 & 1 \end{pmatrix}$ is:

   (a) $\begin{pmatrix} 2 \\ -2 \\ 0 \end{pmatrix}$,

   (b) $\begin{pmatrix} 2 \\ -4 \\ 0 \end{pmatrix}$,

   (c) $\begin{pmatrix} 2 \\ 2 \\ -2 \end{pmatrix}$,

   (d) $\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$,

   (e) None of the above.

7. The angle between the lines

$$\frac{1-x}{2} = \frac{y}{3} = 5 - z \qquad \text{and} \qquad 2x = y = z$$

is:

(a) $\cos^{-1} \dfrac{2}{\sqrt{42}}$,

(b) $\cos^{-1} \dfrac{1}{3\sqrt{14}}$,

(c) $\sin^{-1} \dfrac{1}{\sqrt{14}}$,

(d) $\cos^{-1} \dfrac{2}{3\sqrt{14}}$,

(e) None of the above.

8. Consider the points $A(-1,\,-1,\,2)$ and $B(2,\,1,\,1)$. The intersection point between the line AB and the plane

$$3x + 2y + 5z = 9$$

is:

(a) $\left(-\frac{1}{2},\, 0,\, \frac{3}{2}\right)$,

(b) $\left(\frac{1}{2},\, 1,\, \frac{11}{10}\right)$

(c) $(-1,\, 1,\, 2)$,

(d) $\left(\frac{1}{2},\, 0,\, \frac{3}{2}\right)$,

(e) None of the above.

9. The function $f(x, y) = (x - 3)\ln(\sqrt{xy})$, defined for $x,\ y > 0$, has:

   (a) one saddle,

   (b) one local minimum,

   (c) no stationary points,

   (d) one local maximum,

   (e) None of the above.

10. The solution of the exact differential equation

$$xe^{-2y} + y\cos x + \left(\sin x - x^2 e^{-2y}\right)\frac{dy}{dx} = 0\,,$$

   which satisfies $y = 0$ when $x = 1$, may be written as:

   (a) $x^2 + 2ye^{2y}\sin x = 0$,

   (b) $x^2 + 2ye^{2y}\sin x = e^{2y}$,

   (c) $\sin x - x^2 e^{-2y} = 1$,

   (d) $xe^{-2y} + y\cos x = 0$,

   (e) None of the above.

11. The vector field

$$\boldsymbol{V}(x, y, z) = (\alpha xy - z^3)\boldsymbol{i} + (\alpha - 2)x^2\boldsymbol{j} - (\alpha - 1)xz^2\boldsymbol{k}, \qquad (\alpha \in \mathbb{R}),$$

   is irrotational for:

   (a) $\alpha = 1$,

   (b) $\alpha = 2$,

   (c) $\alpha = 4$,

   (d) $\alpha = 0$,

   (e) None of the above.

12. The directional derivative of the scalar field

$$\phi(x, y, z) = 4e^{2x-y+z}$$

at the point $(1,\ 1,\ -1)$ in a direction toward the point $(-3, 5, 6)$ is:

(a) $\dfrac{20}{9}$,

(b) 0,

(c) $-\dfrac{e^{-5}}{9}$,

(d) $\dfrac{10}{9}$,

(e) None of the above.

SECTION B

13. Find the three roots of $z^3 = 2 + 2i$.
    Plot the three roots, together with $2 + 2i$ on one Argand Diagram. [5 marks]

14. Use Gaussian elimination to find the solution of the system of equations

$$\begin{aligned} 2x_1 + 3x_2 + 4x_3 &= 11, \\ 4x_1 + 3x_2 + 2x_3 &= 13, \\ x_2 + 2x_3 &= 3. \end{aligned}$$

    Briefly explain how your answer would change if the last equation was replaced by $x_2 + 2x_3 = 1$. [10 marks]

15. Given the function $y = f(x) = 48x^2 - x^6$:

    (a) find expressions for $f'(x)$, $f''(x)$; [2 marks]

    (b) find and classify the stationary points of f; [4 marks]

    (c) sketch the curve $y = f(x)$, noting any symmetries it possesses. [4 marks]

16. Find the Taylor expansion of the function

$$f(x,y) = \ln\left(\frac{x^2+y^2}{xy}\right), \qquad x, y > 0,$$

    about the point $(1,1)$, up to and including second-order terms. [7 marks]

17. The second moment of area of a circular sector of angle $2\theta_0$ $(-\theta_0 \leq \theta \leq \theta_0)$ is denoted by I and is given by the formula

$$I = \frac{1}{8}R^4(2\theta_0 - \sin(2\theta_0)),$$

    where R represents the radius of the circular sector. If $\theta_0 = \pi/4$ and the errors in the measurements of R and θ_0 are 0.75% and 1.15%, respectively, find the relative change in I. Express your answer to four significant digits. [8 marks]

18. Consider the vector field

$$\boldsymbol{V}(x,y,z) = \left(2xyz^2,\ x^2z^2 + z\cos(yz),\ 2x^2yz + y\cos(yz)\right).$$

    (a) Show that curl $\boldsymbol{V} = \mathbf{0}$. [3 marks]

    (b) Find the most general form of the scalar field $\phi(x,y,z)$ such that $\boldsymbol{V} = \nabla\phi$. [7 marks]