Maths for Engineers
Class Test Solutions (2018/19)

Q.1 $\vec{AB} = (7,3,3) - (2,1,-2) = (5,2,5) \Rightarrow |\vec{AB}| = \sqrt{54}$

$\vec{AC} = (3,5,-1) - (2,1,-2) = (1,4,1) \Rightarrow |\vec{AC}| = \sqrt{18}$

$$\vec{AB}\cdot\vec{AC} = (5)(1) + (2)(4) + (5)(1) = 18$$

$$\cos\angle BAC = \frac{\vec{AB}\cdot\vec{AC}}{|\vec{AB}||\vec{AC}|} = \frac{18}{\sqrt{18}\sqrt{54}} = \frac{1}{\sqrt{3}} \Rightarrow \angle BAC = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

Hence, correct answer is C

Q.2 $\underline{a}, \underline{b}, \underline{c}$ coplanar $\Leftrightarrow [\underline{a}, \underline{b}, \underline{c}] = 0$

$$\begin{vmatrix} \alpha & 1 & 0 \\ 1 & 2 & 3 \\ -2 & 1 & \alpha \end{vmatrix} = 0 \Leftrightarrow \alpha \begin{vmatrix} 2 & 3 \\ 1 & \alpha \end{vmatrix} - \begin{vmatrix} 1 & 3 \\ -2 & \alpha \end{vmatrix} = 0 \Leftrightarrow \alpha(2\alpha-3) - (\alpha+6) = 0$$

$$2\alpha^2 - 4\alpha - 6 = 0$$

$$\Delta = 16 + 48 = 64 \Rightarrow \alpha_{1,2} = \frac{4 \pm 8}{4} \quad \alpha_1 = 3, \quad \alpha_2 = -1$$

Hence, correct answer is D

Q.3 normal vector is parallel to $\vec{AB} \times \vec{AC}$

$\vec{AB} = (3,1,4) - (-1,2,1) = (4,-1,3)$

$\vec{AC} = (1,1,2) - (-1,2,1) = (2,-1,1)$

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 4 & -1 & 3 \\ 2 & -1 & 1 \end{vmatrix} = \underline{i}(-1+3) - \underline{j}(4-6) + \underline{k}(-4+2) = 2\underline{i} + 2\underline{j} - 2\underline{k} = 2(1,1,-1)$$

$(-1,-1,1) = -\frac{1}{2} \times 2(1,1,-1) \Rightarrow$ Correct answer is A

(Q.4) $f = \frac{x}{yz} + \frac{y}{xz} + \frac{z}{xy}$

$$f_x = \frac{1}{yz} - \frac{y}{x^2 z} - \frac{z}{x^2 y} \Rightarrow f_{xx} = \frac{2y}{x^3 z} + \frac{2z}{x^3 y}$$

$$f_{xx}(1,7,28) = \frac{2\cancel{(7)}}{(1^3)\cancel{(28)}_4} + \frac{2\cancel{(28)}^4}{(1^3)\cancel{(7)}} = \frac{1}{2} + 8 = \frac{17}{2} \Rightarrow$$ Correct answer is (B)

(Q.5) $u = xy \Rightarrow \frac{\partial u}{\partial x} = y \quad \frac{\partial u}{\partial y} = x$

$v = \frac{x}{y} \Rightarrow \frac{\partial v}{\partial x} = \frac{1}{y} \quad \frac{\partial v}{\partial y} = -\frac{x}{y^2}$

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial u}\frac{\partial u}{\partial x} + \frac{\partial f}{\partial v}\frac{\partial v}{\partial x} = y\frac{\partial f}{\partial u} + \frac{1}{y}\frac{\partial f}{\partial v}$$

$$\frac{\partial f}{\partial y} = \frac{\partial f}{\partial u}\frac{\partial u}{\partial y} + \frac{\partial f}{\partial v}\frac{\partial v}{\partial y} = x\frac{\partial f}{\partial u} - \frac{x}{y^2}\frac{\partial f}{\partial v}$$

Hence $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = xy\frac{\partial f}{\partial u} + \cancel{\frac{x}{y}\frac{\partial f}{\partial v}} + xy\frac{\partial f}{\partial u} - \cancel{\frac{x}{y}\frac{\partial f}{\partial v}} = 2xy\frac{\partial f}{\partial u}$

PDE becomes $2xy\frac{\partial f}{\partial u} = 2x^2\sin(xy) \Rightarrow \frac{\partial f}{\partial u} = v\sin(u)$

Correct answer is (B)

(Q.6) $T = \underbrace{\frac{2\pi}{\sqrt{G}}}_{\text{constant}} \frac{R^{3/2}}{M^{1/2}} = K\left(\frac{R^{3/2}}{M^{1/2}}\right) \equiv f(R, M)$

$$f_R = \frac{3}{2}K\left(\frac{R^{1/2}}{M^{1/2}}\right) \qquad f_M = -\frac{1}{2}K\left(\frac{R^{3/2}}{M^{3/2}}\right)$$

$$\varepsilon_T = \underbrace{\left(\frac{R_o f_R^o}{f_o}\right)}_{\frac{3}{2}}\varepsilon_R + \underbrace{\left(\frac{M_o f_M^o}{f_o}\right)}_{-\frac{1}{2}}\varepsilon_M$$

R_o, M_o = reference values

$\varepsilon_R = +0.5\%$ $\quad \varepsilon_M = -1.25\%$

$\hookrightarrow \varepsilon_T = (1.5)(0.5\%) + (0.5)(1.25\%) = 1.375\%$

Correct answer is (A)

Q.7 $f = e^{y^2}\cos(x-1)$ $\quad (1,0)$

$f_x = -e^{y^2}\sin(x-1)$

$f_y = 2y\,e^{y^2}\cos(x-1)$

$f_{xx} = -e^{y^2}\cos(x-1)$

$f_{yy} = 2e^{y^2}\cos(x-1) + 4y^2 e^{y^2}\cos(x-1)$

$f_{xy} = -2y\,e^{y^2}\sin(x-1)$

$f_0 = 1$

$f_x^0 = 0$

$f_y^0 = 0$

$f_{xx}^0 = -1$

$f_{yy}^0 = 2$

$f_{xy}^0 = 0$

$f = 1 - \frac{1}{2}(x-1)^2 + y^2 + \ldots$

Correct answer is D

$y = ax+b$ line of best fit

Q.8 $f(a,b) = (1-b)^2 + (3a-b)^2 + (3-2a-b)^2$

$f_a = 0 \Rightarrow 13a - b = 6$

$f_b = 0 \Rightarrow -a + 3b = 4$

2 eqns in 2 unknowns $\Rightarrow a = \frac{22}{38}$

Correct answer is A

Q.9 $(y-3)\frac{dy}{dx} = -x \Rightarrow (y-3)\,dy = -x\,dx \Rightarrow \frac{1}{2}(y-3)^2 = -\frac{x^2}{2} + C$

$\Rightarrow y^2 - 6y = -x^2 + K \quad (K \in \mathbb{R})$

$y=1$ when $x=0 \Rightarrow K=-5 \Rightarrow y^2 - 6y + (x^2+5) = 0$

Solve quadratic in x: $y = 3 \pm \sqrt{4-x^2} \Rightarrow y = 3 - \sqrt{4-x^2}$

the plus sign violates the initial cond.

Correct answer B

Q.10 $x^2(x+2)\frac{dy}{dx} - x^3 y = 2x \Rightarrow \frac{dy}{dx} - \left(\frac{x}{x+2}\right)y = \frac{2}{x(x+2)} \Rightarrow P(x) = -\frac{x}{x+2}$

I.F. $= \exp\left(\int P(x)dx\right)$ $\quad \int P(x)dx = -\int\left(\frac{x+2}{x+2} - \frac{2}{x+2}\right)dx = -x + 2\ln(x+2)$

$= -x + \ln(x+2)^2$

I.F. $= e^{-x+\ln(x+2)^2} = e^{-x}\cdot e^{\ln(x+2)^2} = e^{-x}(x+2)^2 \Rightarrow$ correct answer E