School of Mathematics and Statistics

GUIDE

for Students & Advisers

2010–11

Honours Degrees in

MATHEMATICS

Contents

Introduction

If you are now approaching the end of your Level 2 Mathematics courses then it is time for you to think about what you want to do at Levels 3, 4 and 5.

We in the School of Mathematics and Statistics hope that you will continue to study with us. We offer a range of Honours Degrees involving Mathematics. We also have some Level 3 non-Honours courses for those of you wishing to complete a Designated Degree.

This guide gives a brief guide to our Honours Degrees involving Mathematics. Further details of our Honours Degrees, details of our Level 3 non-Honours courses, and details of all our courses can be found on our Moodle pages. Details of the Honours Degrees involving Statistics are available in a separate guide.

It should be noted that, although the contents of this guide are accurate at the time of printing, the courses on offer are subject to change at any time as a response to circumstances.

If you have any queries that are not answered in this guide, staff in the School of Mathematics and Statistics will be happy to help. Contact details are included on the next page.

Printed versions of this guide can be obtained from the General Office of the School of Mathematics and Statistics, currently Room 328, Tel: 0141 330 5176.

The Honours Programme

Our Honours and M.Sci. Degrees

We offer Honours and M.Sci. Degrees in

- Mathematics,
- Pure Mathematics,
- Applied Mathematics,
- Mathematical Sciences (nb not available as an M.Sci. Degree).

Honours Degrees (all B.Sc. and M.A. programmes) involve a further 2 years of study (4 years in total); M.Sci. Degrees (all M.Sci. programmes) involve a further 3 years of study (5 years in total). Entry into these degree programmes is at Level 3, and is subject to meeting certain requirements that are fully explained later in this chapter.

Honours Degrees in **Mathematics**, **Pure Mathematics** and **Applied Mathematics** are available as:

- *Single Honours Degrees* or *Combined Honours Degrees* with another subject (available with a wide range of other subjects; recently introduced combinations are Accounting + Maths/Pure Maths/Applied Maths, and Finance + Maths/Pure Maths/Applied Maths);
- *B.Sc. M.A.* or *M.Sci. Degrees*. (The M.Sci. Degree is not available in the Faculty of Arts and the Faculty of Law, Business and Social Sciences.)

The *M.Sci. Degrees* are more challenging than the B.Sc. and M.A. Degrees, and involve additional components. Full details on our *M.Sci. Degrees* are given in the next chapter.

Our *Honours Degree in Mathematical Sciences* is an integrated B.Sc. Degree comprising Computing Science, Mathematics and Statistics. It is run jointly with the School of Computing Science.

Should I choose Applied Mathematics, Pure Mathematics or Mathematics?

Applied Mathematics arises in an effort to solve various real-life problems. The Applied Mathematics degree programme allows students with a flair for mathematics, who prefer the practical and applicable aspects of the subject, to concentrate on these elements at Honours level.

Pure Mathematics is usually abstract in nature, building via definitions, theorems and proofs. The Pure Mathematics degree programme is ideal for those students who prefer the abstract and logical aspects of the subject.

The Mathematics degree programme allows you complete choice across the entire spectrum of our mathematical courses.

Our normal advice to students would be to choose Mathematics, since, as will be seen in the section on Course Options, this gives students the greatest amount of choice. It is unwise to assume that, because you did not like a particular course in Level 2, you will not like courses with the same flavour in Levels 3 and 4.

We also offer the options of an Applied Mathematics Degree or a Pure Mathematics Degree to those students who feel that those degrees fit better with their long-term plans.

Abbreviations used later in this booklet:

Courses leading to Honours Degrees completed over 4 years:

3H	B.Sc. or M.A. Honours in Mathematics as a single subject;
3HA	B.Sc. or M.A. Honours in Applied Mathematics as a single subject;
3HP	B.Sc. or M.A. Honours in Pure Mathematics as a single subject;
3CH	B.Sc. or M.A. Honours in Mathematics combined with another subject;
3CHA	B.Sc. or M.A. Honours in Applied Mathematics combined with another subject;
3CHP	B.Sc. or M.A. Honours in Pure Mathematics combined with another subject;
3HS	B.Sc. in Mathematical Sciences (Computer Science, Mathematics, Statistics).

Courses leading to Honours Degrees completed over 5 years:

3M	M.Sci. in Mathematics as a single subject;
3MA	M.Sci. in Applied Mathematics as a single subject;
3MP	M.Sci. in Pure Mathematics as a single subject;
3CM	M.Sci. in Mathematics combined with another subject*
3CMA	M.Sci. in Applied Mathematics combined with another subject*.
3CMP	M.Sci. in Pure Mathematics combined with another subject*.

* See the next chapter for details.

Coordinators

All enquiries concerning our Honours Degrees should be directed to

Dr. C. Coman (Level 3 Coordinator)
Room 504, Tel. 0141 330 5179
E-mail: c.coman@maths.gla.ac.uk

Prof. S.J. Pride (Level 4 Coordinator)
Room 431, Tel. 0141 330 6528
E-mail: s.pride@maths.gla.ac.uk

Dr. A.J. Baker (Level 5 Coordinator)
Room 403, Tel. 0141 330 6140
E-mail: a.baker@maths.gla.ac.uk

General Information

Lecture Courses: Each lecture course at Levels 3 and 4 comprises 25 lectures which take place either on Mondays, Tuesdays and alternate Wednesdays or on alternate Wednesdays, Thursdays and Fridays, covering 10 weeks of the semester. Each lecture course has a value of 15 credits.

Tutorials: Lecturers will explain the tutorial arrangements for their courses during the first week of lectures. Typically there will be 9 tutorials associated with each lecture course, and your attendance is compulsory.

Private Study: You should expect to study between 4 and 6 hours each week per lecture course in addition to the 34 scheduled contact hours. If you find you need more than 6 hours then consult your lecturer in the first instance for advice. If you do less than 3 hours each week of private study on a course, then you will almost certainly not do yourself justice in the examination assessment.

Provision for Personal Development: At Level 3 we provide a range of activities that contribute to your Personal Development Plan (PDP). These activities include:

- presentations and seminars (which give you the opportunity to work in small groups on short projects and present your work to others);
- an introduction to LaTeX (the most commonly used software for typesetting mathematics);
- opportunities to use powerful mathematical software such as Mathematica.
- employment related presentations relating to interview techniques and the writing of curriculum vitae.

The first three of these items will be components of the Semester 2 course Writing and Presenting Mathematics.

Level 4 Project: At Level 4 students are required to undertake an individual 20 credit project which is expected to involve proportionately more work and time than a typical 15 credit lecture course. As preparation for their projects, students will take the Level 3 course in Writing and Presenting Mathematics. Students on a combined honours programme may choose to do their project either in Mathematics or in their other subject area: a combined honours student who will take their project in their other subject is not required to take Writing and Presenting Mathematics.

Note that Degree Regulations require all students to complete a piece of independent work as part of their Degree Programme. This can, in Mathematics, be either a standard project or participation in the Ambassador Scheme. **Starting from 2010-11 the University is interpreting the word 'complete' to mean 'obtain at least grade D in'.** So, for any student taking a project in Mathematics, it is essential that you treat the Project seriously in order to give yourself the best possible chance of obtaining at least a grade D in it.

Level 5 Project: The project at Level 5 is a substantial piece of work valued at 45 credits. A detailed description of the project requirements, objectives and length, etc is given on the Mathematics Moodle webpages.

Course Options

All our Honours Degrees require you to select lecture courses and other activities from our Honours programme. Different selection rules apply to different Honours Degrees. This information is contained in Tables 1 and 2.

Detailed descriptions of all courses can be obtained by going to the Mathematics Moodle pages. Brief descriptions of Level 3 and Level 4 courses are given in the appendices to this booklet.

Table 1: **Level 3 Honours Programme - list of lecture courses & selection rules**

	3H 3M	3HP 3MP	3HA 3MA	3CH 3CM	3CHP 3CMP	3CHA 3CMA	3HS
Semester 1							
(P) Geom and Topology I	2	1	1	2	2		
(P) Algebra I	2	C	1	2	2		1
(M) Analysis I	C	C	C	2	2	1	1
(M) Mathematical Methods I	C	C	C	2	2	C	C
(A) Dynamics I	2	1	C	2		1^*	
(A) Topics from Applied Mathematics (Mathematical Ecol 2010/Biol 2011)	2	1	1	2		$1^\dagger$	
Semester 2							
(P) Geometry and Topology II	2	C		1/2	1/2	1/2	
(P) Algebra II	2	C		1/2	1/2		$1^\ddagger$
(M) Analysis II	C	C	C	1/2	1/2	1/2	$1^\ddagger$
(M) Writing and Presenting Mathematics	C	C	C	$\widehat{C}$	$\widehat{C}$	$\widehat{C}$	$\widehat{C}$
(A) Mathematical Methods II	2		C	1/2	1/2	1/2	$1^\ddagger$
(A) Mathematical Modelling I	2		C	1/2	1/2	1/2	

C - A compulsory course
$\widehat{C}$ - compulsory for students intending to take a project in Mathematics in Level 4
n - Choose n courses from the available choices in the semester
* - Not available for students taking Combined Honours with Physics
† - Compulsory course for Combined honours with Zoology at Levels 3 and 4
‡ - students not taking Writing and Presenting Mathematics choose 1
1/2 - students not taking Writing and Presenting Mathematics choose 2, all other students choose 1.

Note that Single Honours Applied Mathematics students should, over the two years of their Honours programme, take at least 10 courses from those marked A and M, with a minimum of 7 courses from those marked A. Similarly Pure Mathematics students should take at least 10 courses from those marked M and P, with a minimum of 7 courses from those marked P.

Combined Honours Applied Mathematics students should, over the two years of their Honours programme, take at least 4 courses from those marked A and M, if taking a project in Mathematics, and should take at least 5 courses from those marked A and M otherwise. Similarly Pure Mathematics students should take at least 4 courses from those marked M and P, if taking a project in Mathematics, and should take at least 5 courses from those marked M and P otherwise.

The courses are listed in Tables 1 and 2 in streams (e.g. Analysis I, II, III and IV). The normal rule is that an earlier course in a stream is a pre-requisite for the later courses in that stream. The exceptions to this are that Geometry & Topology I is *not* a pre-requisite for any later Geometry & Topology courses, that the Numerical Analysis courses are independent of one another as are the Topics from Applied Mathematics courses and the Topics from Pure Mathematics courses. In contrast, Mathematical Methods I *is* a pre-requisite for all the following Mathematical Methods *and* Mathematical Modelling courses.

We would normally expect that a Single Honours student would choose four streams at the start of Level 3 and would then continue with those streams throughout the remainder of Level 3 and Level 4, possibly choosing in Level 4 to start on some of the new streams taking place there. Combined Honours students would choose two streams at the beginning of Level 3 and then follow a similar trajectory.

Choice of courses: This is subject to approval by the Head of the School of Mathematics and Statistics. If in any term you have a choice of courses, you are *not* required to declare your choices at the start of term. You may, if you wish, sample the alternatives for a week or two before choosing.

Combined Honours Students: The courses described in Table 1 represent approximately 50% of your total workload, the other 50% coming from the other subject. Combined Honours students should be aware that

Mathematical Methods I is a prerequisite for all courses in the Mathematical Methods and Mathematical Modelling streams. Combined Honours Applied Mathematics students are therefore very strongly urged to take this course.

Mathematical Sciences Students: The above courses represent approximately 33% of your total workload, the other 67% coming from Computing Science and Statistics.

Level 3 courses in Level 4: Combined honours students may take ONE Level 3 course in Level 4 and single honours students may take TWO Level 3 courses at Level 4. Moreover, Topics from Applied Mathematics I (which will be Mathematical Biology in 2011–12) may be taken at Level 4 without prejudice to this allowance. Students in Level 4 are *not* allowed to take Writing and Presenting Mathematics as one of their courses. Note that a Level 3 course taken at Level 4 is assessed separately from the same course taken at Level 3.

Selection Rules for the Level 4 Honours Programme			
	Maths, Pure Maths & Applied Maths		**Math Sciences**
	Single Honours Maths (B.Sc. /M.A./M.Sci.)	Combined Honours (B.Sc/M.A./M.Sci.)	B.Sc.
Semester 1	3 courses+project	1 course+project or 2 courses (if project is in other subject)	1 course+project or 1/2 courses (if project is in other subject)
Semester 2	4 courses	2 courses	1 course or 2/1 courses (if project is in other subject)

Important: Some courses have prerequisite courses, which may restrict your choice. For details see the Level 4 Honours information on Moodle.

Table 2: **Level 4 Honours programme in 2010–2011 - list of courses**

	4H 4M	4HP 4MP	4HA 4MA	4CH 4CM	4CHP 4CMP	4CHA 4CMA	4HS
Semester 1							
Project	C	C	C	C*	C*	C*	C*
(P) Geometry and Topology III (Differential Geometry)	3	3	3	1/2*	1/2*	1/2*	
(P) Analysis III (Linear Analysis)	3	3	3	1/2*	1/2*	1/2*	2/3*
(P) Algebra III (Galois Theory)	3	3	3	1/2*	1/2*		2/3*
(M) Numerical Analysis 1 (Numerical Analysis)	3	3	3	1/2*	1/2*	1/2*	2/3*
(A) Topics from Applied Mathematics I (Mathematical Ecol 2010/Biol 2011)	3	3	3	1/2*	1/2*	1/2*†	2/3*
(A) Mathematical Methods III (Nonlinear Waves)	3	3	3	1/2*	1/2*	1/2*	2/3*
(A) Mathematical Modelling II (Mathematical Modelling II)	3	3	3	1/2*	1/2*	1/2*	
Semester 2							
(P) Geometry and Topology IV (Algebraic Topology)	4	4	4	2	2	2	
(P) Analysis IV (Integration)	4	4	4	2	2	2	2/3*
(P) Topic from Pure Mathematics I (Number Theory)	4	4	4	2	2	2	2/3*
(M) Topic from Pure Mathematics II (Discrete Mathematics)	4	4	4	2	2	2	2/3*
(M) Topic from Applied Mathematics II (Financial Mathematics ‡)	4	4	4	2	2	2	2/3*
(M) Numerical Analysis II (NSPDEs)	4	4	4	2	2	2	2/3*
(A) Dynamics II (Hamiltonian Mechanics)	4	4	4	2		2	
(A) Mathematical Modelling III (Mathematical Modelling III)	4	4	4	2	2	2	

See the next page for the key to the above table.

C - The project is compulsory for Single Honours students

C* - Combined Honours students must do a project in Mathematics or in their other subject

1/2* - Combined Honours students choose either a project in Mathematics and one course or a project in their other subject and two Mathematics courses

n - Choose *n* courses from the available choices

† - Compulsory course for Combined Honours with Zoology at Level 4 but otherwise optional

2/3* - Mathematical Sciences students choose either a project in Mathematics and one course from each semester or 3 courses with at least one course from each semester.

‡ students studying Combined Honours with Statistics may not take both Financial Mathematics and Financial Statistics.

Entry Requirements

These are stated precisely below. Key points to note are:

! In exceptional circumstances the Head of the School of Mathematics and Statistics may permit a student who does not meet the entry requirements to enter a course.

! Faculty regulations need to be satisfied in order to enter honours programmes. Please check faculty regulations with your Adviser of Studies.

Students must satisfy the regulations for continuation in the faculty in which they study. Mathematics impose the following additional prerequisite conditions.

Single and Combined Honours in Mathematics (3H/3CH), Pure Mathematics (3HP/3CHP) or Applied Mathematics (3HA/3CHA): The Level 2 Honours package (2A, 2B, 2C, 2D, 2E, 2F) with a minimum grade of D in each module and a GPA of 12 or better, normally at the first attempt. Combined Honours students must also satisfy the entry requirements for their other subject.

Single and Combined MSci. in Mathematics (3M/3CM), Pure Mathematics (3MP/3CMP) or Applied Mathematics (3MA/3CMA): The Level 2 Honours package (2A, 2B, 2C, 2D, 2E, 2F) with a minimum grade of C in each module and a GPA of 14 or better, normally at the first attempt. Combined Honours students must also satisfy the entry requirements for their other subject.

Honours Mathematical Sciences - 3HS: The Algebra/Calculus package (2A, 2B, 2D) and 2E, 2F with a minimum grade of D in each module and a GPA of 12 or better, normally at the first attempt. Students must also have taken Statistics - 2R, 2S, 2X and 2Y; and Computing Science courses Java Programming 2, Algorithmic Foundations 2, Algorithms & Data Structures 2 and Object-Oriented Software Engineering 2, with an overall average of C or better. Students must also satisfy the entry requirements for Statistics and Computing Science.

Examinations

We hold examinations at the end of Level 3 on your Level 3 work, and at the end of Level 4 on your Level 4 work. There is a separate examination paper for each lecture course. The final mark for each year is calculated by combining course marks and the project mark (if appropriate) in proportion to their credit ratings.

At the end of Level 3 you are given grades for each course based on the Level 3 diet of examinations. For *Mathematics*, *Pure Mathematics* and *Applied Mathematics Degrees* you must obtain an overall average of at least grade D to continue to Level 4 B.Sc. /M.A. For the *Mathematical Sciences Degree* you must obtain an overall average of at least grade C from the three subjects to continue.

At the end of Level 4 your Honours Degree classification in Mathematics is based on both Level 3 and Level 4 examinations, Level 3 counting as 40% of the total assessment and Level 4 as 60% of the total assessment

You should note that, to maintain the integrity of the examination system, our External Examiners are insistent that we should fully implement the requirements for Absence Reporting when students miss examinations. These are to be found under Absence Reporting Guidelines on the Absence Records page of WebSurf. Specifically these require that a student with a Significant Absence (which includes any absence where an examination paper is missed) for a medical reason is encouraged to submit medical evidence where it is available. Any student with a **Significant Absence** for any reason whatsoever **must supply Other Evidence**, which can include a note from the police, a note from a hospital, a letter from a student counsellor or other professional, a note from an independent responsible person who can vouch for the event that led to the absence, or evidence from a member of staff who was alerted to the circumstances at the time. Failure to abide by these Guidelines will lead to the award of grade 7 (deferred result) which will only alter once the Guidelines haver been complied with.

Further details (including formal regulations, course-work requirements and the prizes we award) are on the Mathematics Moodle pages.

The M.Sci. Degrees

We offer the following Degrees:

- *M.Sci. Degrees in Mathematics*, *Pure Mathematics* or *Applied Mathematics*.
- *Combined M.Sci. Degrees in Mathematics*, *Pure Mathematics* or *Applied Mathematics*. Available with Astronomy, Chemistry, Computing, Physics and Statistics.

Entry into the M.Sci. Programme is at Level 3. **Prerequisites** are a grade C or better in each component of the Level 2 Honours Package (i.e. 2A, 2B, 2C, 2D, 2E and 2F), with a grade point average (GPA) of at least 14 over all these courses, normally at the first attempt. The M.Sci. Degree is not available to students enrolled in the Faculty of Arts or the Faculty of Law, Business and Social Sciences.

The structure and features of our M.Sci. Programme can be seen over the next few pages; see *Tables 3* and *4* as well as the list of SMSTC (*Scottish Mathematical Sciences Training Centre*) courses available. Important points:

1. At the end of Level 3 students may take a Designated Degree;
2. At the end of each of Level 3 and Level 4 you are given grades based on that diet of examinations. The requirement for continuation at Level 3 and at Level 4 is an overall average of grade B or better, at that diet.
3. The students in Single Honours Degrees have the following "exit" options:
   (a) after Level 3, students may proceed to Level 4 of the B.Sc. (Hons.) Mathematics or Pure Mathematics or Applied Mathematics as appropriate;

(b) after Level 4, students will normally be able to graduate B.Sc. (Hons.) in Mathematics or Pure Mathematics or Applied Mathematics as appropriate.

4. Students may take one SMSTC course or two half-SMSTC courses instead of two Level 5 courses. Combined M.Sci. students may take one half-SMSTC-course instead of one Level 5 course (all subject to approval). Tutorial support will be provided by the School for the approved courses.

Table 3: **Provisional List of Mathematics, Pure Mathematics and Applied Mathematics courses at Level 3, Level 4 and Level 5 available to students entering Level 3 honours programmes in session 2010-11.**

Level 3	**Level 4**	**Level 5**
Geometry & Topology I	Algebraic Topology	Advanced Algebraic Topology*
Algebra I	Differential Geometry	Advanced Group Theory*
Analysis I	Discrete Mathematics	Algebraic Geometry & Commutative Algebra*
Mathematical Methods I	Financial Mathematics	Advanced Linear Elasticity
Dynamics I	Galois Theory	Banach Algebras
Topics from Appl. Mathematics I (Mathematical Ecology)	Hamiltonian Mechanics	Biological & Physiological Fluid Mechanics*
Geometry & Topology II	Integration	Category Theory
Algebra II	Linear Analysis	Complex Algebraic Curves
Analysis II	Mathematical Biology	Complex Analysis 2*
Writing & Presenting Mathematics	Mathematical Modelling II	Elasticity
Mathematical Methods II	Mathematical Modelling III	Fluid-Structure Interactions
Mathematical Modelling I	Nonlinear Waves	Magnetohydrodynamics
	Number Theory	Modern Number Theory & Cryptography
	Numerical Analysis	Ring & Representation Theory
	NSPDEs	Solitons*
	Project	Special Relativity & Classical Theory of Fields*
		Special Topics in & Continuum Physics & Materials
		Stochastic Differential Equations

Note: Options marked with * are those courses available in 2010/2011. There will be a maximum of 3 other courses also offered in 2010/11, subject to staff availability.

Prerequisites

Table 4: **Prerequisites for Level 5 courses**

Name of Course	**Level**	**Pre-requisites**
Advanced Group Theory	5	*Introductory Algebra, Groups, Rings & Fields* or Algebra I, II
Algebraic Geometry and Commutative Algebra	5	*Galois Theory,* or Algebra III
Advanced Linear Elasticity	5	*Continuum Mechanics, Introduction to Linear Elasticity* or Mathematical Modelling II
Biological and Physiological Fluid Dynamics	5	*Fluid Dynamics* or Mathematical Modelling III
Category Theory	5	*None (however, many examples will be taken from abstract algebra and topology, so an acquaintance with those areas will be an advantage)*
Complex Algebraic Curves	5	*None*
Complex Analysis 2	5	*Complex Analysis 1* or Analysis II
Elasticity	5	*Continuum Mechanics* or Mathematical Modelling II
Fluid-Structure Interactions	5	*Fluid Dynamics* or Mathematical Modelling III or some basic knowledge of Solid and Fluid Mechanics
Magnetohydrodynamics	5	*Fluid Dynamics* or Mathematical Modelling III
Modern Number Theory and Cryptography	5	*Discrete Mathematics* or Algebra I, II
Ring & Representation Theory	5	*Groups, Rings & Fields* or Algebra I, II
Solitons	5	*Differential Equations 1&2, Nonlinear Waves* or Mathematical Methods I, II & Nonlinear Waves
Special Relativity and Classical Theory of Fields	5	*Newtonian Mechanics, Differential Equations 1 & 2, Calculus of Variations* or Mathematical Methods I, II & Dynamics I
Stochastic Differential Equations	5	*Probability* or some knowledge of Probability

Combined M.Sci. students: the courses chosen from those in Table 3 represent 50% of your total workload, the other 50% coming from the other subject.

Examinations: Students must take end of course examinations for each of the 15 credit courses that they are required to complete. Single Honours MSci students are also required to do a Level 5 project which is worth 45 credits. Combined Honours MSci can, in some cases, choose whether to take their project in Mathematics or the other subject – detailed information can be obtained from the *Level 5 Course Coordinator*. There is a wide variety of project topics available – further information can be obtained from the *Level 5 Course Coordinator* (see page 6 for contact details).

At the end of Level 5 your Honours Degree classification is based on Level 3, Level 4 and Level 5 examinations, the contributions from Levels 3, 4 and 5 being 24%, 36% and 40%, respectively.

You should note the paragraph on Significant Absences in the Examinations section of the Honours Programme chapter.

SMSTC Courses: The official website for the SMSTC courses can be found at

`http://www.smstc.ac.uk`

and contains more information and details of the courses on offer. All these courses aim to give an introduction to a range of important higher-level topics, and build on courses given at Levels 3 and 4. SMSTC courses are delivered live via a weblink by experts in their fields from other Universities. Tutorial support will be provided by members of the Mathematics section of the School of Mathematics and Statistics. Only those SMSTC courses specified below are being offered in 2010-11. These courses are available in addition to those listed in Table 3. A brief description is given below; listed for each course you will find the topics that you should be familiar with by the end of the corresponding course.

SEMESTER 1

Advanced Group and Module Theory 1 (SMSTC)

- basic concepts in group theory, including definitions and examples; constructions of groups; generators and relations; simple groups; the Jordan-Hölder theorem; soluble groups; group actions; conjugation; Sylow theorems and applications;
- the definitions and basic properties of rings and modules; chain conditions; Hilbert basis theorem; PIDs, Euclidean domains and UFDs.

Applied Analysis 1 (SMSTC)

- dynamical systems and bifurcation;
- systems of hyperbolic PDEs and shock waves.

Advanced Methods of Applied Mathematics 1 (SMSTC) (available in 2010 – 11)

- asymptotic methods for differential equations (including multiple scales and boundary layers, singular perturbations, matched asymptotics);
- contour integral methods for differential equations.

Advanced Geometry and Topology 1 (SMSTC)

- differentiable manifolds and transversality;
- the fundamental group and covering spaces;
- differential forms and de Rham cohomology.

Advanced Mathematical Modelling 1 (SMSTC)

- traffic flow;
- continuum mechanics (notation, theory and examples including Newtonian and non-Newtonian fluids, solids).

Pure Analysis 1 (SMSTC)

- measure and integration;
- concrete examples (Riemann and Lebesgue);
- abstract theory – convergence theorems;
- complex, product and Radon measures;
- fractal sets and Hausdorff dimension;
- L^p spaces and Differentiation.

SEMESTER 2

Advanced Group and Module Theory 2 (SMSTC)

- some elementary background on algebraic numbers and algebraic integers; finitely generated modules over a PID; Jordan canonical form of a matrix; the Artin-Wedderburn theorem; modules over semi-simple Artinian rings;
- Maschke's theorem; characters and character tables; tensor products; applications to groups such as Burnside's paqb-theorem.

Applied Analysis 2 (SMSTC)

- analysis of parabolic and elliptic PDEs (including existence, uniqueness, maximum principles, energy estimates, monotone iteration, regularity, eigenfunctions);
- Sobolev embedding and trace theorems;
- finite element approximations.

Advanced Methods of Applied Mathematics 2 (SMSTC)

- numerical methods (including IVPs, BVPs, numerical linear algebra, practical optimisation).

Advanced Geometry and Topology 2 (SMSTC)

- vector bundles;
- Riemannian metrics;
- connections and curvature;
- the Hodge Laplacian and harmonic form.

Advanced Mathematical Modelling 2 (SMSTC)

- continuum mechanics (magnetohydrodynamics);
- molecular dynamics;
- mathematical biology (ecology, disease evolution, waves, cardiac propagation, developmental biology, cancer modelling).

Pure Analysis 2 (SMSTC)

- functional analysis;
- Banach and Hilbert spaces;
- Baire Category, Open Mapping and Uniform Boundedness Principles;
- weak and weak* topologies;
- compact operators;
- spectral theory (C* - algebras).

Further Information

We hope you take our courses because you enjoy them. But naturally you also choose your courses with an eye to your future. We draw your attention to the following points.

Research Opportunities: We, as a School, are very active in research and are always keen to recruit high-quality graduates as postgraduate students. For details of our current research see

`http://www.maths.gla.ac.uk/research/`

We also hope to provide some of you with an opportunity to sample mathematics research in the summer vacation between Levels 3 and 4. Details will be circulated at the start of Semester 2 of Level 3.

Career Opportunities: Mathematics graduates are in great demand by a wide range of employers, notably in the financial, insurance, information technology and engineering sectors. But don't just take our word for it: see

`http://www.maths.gla.ac.uk/undergrad/careers.htm`

Appendix 1 - Level 3 Courses

ALGEBRA I The basics of group theory and ring theory will be covered. The intention for the course is two-fold: that it should stand on its own as a good introduction to the ideas and methods of abstract algebra; and that it should combine with the Algebra II course to form a substantial grounding in the subject.

Students attending the course are expected to acquire a good working knowledge of the basics of group theory and ring theory, and to develop skills in thinking logically, formulating precise mathematical arguments, solving problems and presenting solutions in a good mathematical style. By the end of the course students should: (a) be familiar with the definitions, concepts, results and methods of proof relating to each part of the syllabus; (b) be able to quote the definitions and results, and to reproduce the proofs of some key results; (c) be able to solve problems relating to the material covered. These might be straightforward applications of the definitions and results but might also be problems of a more testing nature.

ALGEBRA II By the end of this course students will have covered all the principal core topics of abstract algebra and should have the appropriate level of knowledge for undertaking any further algebra course of their choice.

By means of lectures, tutorials and private study students are expected to (a) acquire a good working knowledge of the basics of group theory and ring theory; (b) develop their skills in thinking logically, formulating precise mathematical arguments, solving problems and presenting solutions in a good mathematical style. More specifically, by the end of the course students should be (a) familiar with the definitions, concepts, results and methods of proof relating to each section of the syllabus; (b) able to quote these definitions and results, and to reproduce the proofs of some key results; (c) able to solve a range of problems relating to the material covered.

ANALYSIS I The principal aim of this course is to develop the ability to construct rigorous mathematical arguments in the context of real analysis. The thread underlying the content of the course is the notion of completeness for the real numbers. The course will examine this notion in detail, using it to study sequences, series and functions on the real line. The course will use completeness to give a rigorous account of differentiable functions, power series and their properties, introduce the notion of uniform continuity and develop a theory of integration for continuous functions on closed and bounded intervals.

By means of lectures, tutorials and private study students should be able to (a) state the completeness axiom for the real line using Existence of Suprema, Limits of bounded monotone sequences, The Bolzano-Weierstrass property, Cauchy Sequences, The intermediate value theorem; (b) relate the various forms of the completeness axiom to one another with a rigorous proof and be able to use the appropriate version of the completeness axiom to solve elementary problems in real analysis; (c) explain the difference between continuity and uniform continuity, determine in concrete cases whether a given function is uniformly continuous and use completeness to show that continuous functions on closed and bounded intervals are uniformly continuous; (d) show from the definition that a given function is differentiable and prove some basic properties of differentiable functions, be able to use the Mean value theorem and Taylor's theorem; (e) determine the radius of convergence of a power series and use power series to define functions; (f) explain how to define the integral of a continuous function on a closed and bounded interval and be able to state, prove and apply the fundamental theorem of calculus.

ANALYSIS II Analysis II will begin by revising and consolidating the theory of complex numbers. Basic concepts of complex analysis such as complex differentiability and analytic functions will be introduced. Elementary properties of analytical functions will be studied and fundamental results governing their behaviour (Cauchy's and Taylor's theorems) will be proved. Cauchy's Residue theorem will be proved and techniques of contour integration (contour integration provides a very powerful method for evaluating otherwise intractable improper real integrals) will be developed.

By means of lectures, tutorials and private study students should be able to (a) determine whether a given function of a complex variable is analytic; (b) find the power series expansion of an analytical function about a given point in its domain of definition; (c) find the Laurent expansion of a meromorphic function about any of its poles, and determine the residues at such points; (d) evaluate a reasonable selection of contour integrals using the method of residues, and apply contour integration to the evaluation of improper real integrals.

DYNAMICS I The aim of Dynamics I is to build on the introduction to the Newtonian mechanics of particles provided in 2C. Finite systems of interacting particles will be considered. Rigid body dynamics will be developed, Euler's equations will be established and the Lagrangian formulation of Newtonian Mechanics will be formulated and illustrated for symmetric and asymmetric tops.

By means of lectures, tutorials and private study students should be able to (a) handle two dimensional problems involving central forces; (b) derive the energy conservation equation from the equation of a motion of a particle; (c) understand stability and be able to show when orbits are stable and whether equilibrium points are stable or not stable; (d) understand rotating and accelerating frames of reference; know and understand the basic laws of mechanics and be able to apply them in a variety of situations; (e) compute the inertia tensor for various simple rigid bodies; (f) construct the Lagrangian function for simple systems and use it to obtain the equations of motion; (g) understand Euler angles and be able to use them to analyse the motion of spinning symmetric and asymmetric tops.

GEOMETRY AND TOPOLOGY I The course looks at geometry in the way suggested by Felix Klein in his Erlanger lecture. It discusses five geometries and the relationship between them.

By means of lectures, tutorials and private study students should be able to (a) understand the axiomatic approach to mathematics; (b) explain the Klein view of geometry and the projective hierarchy; (c) use the Klein philosophy to solve problems in euclidean geometry; (d) define the five geometries (euclidean, affine, projective, inversive and hyperbolic) and their basic properties; (e) compute in the groups of the various geometries; (f) understand and apply the Fundamental Theorem of each geometry; (g) prove the main theorems in the notes, and use these results to solve other problems.

GEOMETRY AND TOPOLOGY II The course is concerned with extending the basic concepts and techniques of analysis on the real line to the more general setting of metric spaces, and illustrating the power of the resulting theory by giving applications to such areas as differential equations.

Students will be expected to master the basic ideas treated in the syllabus, and to develop an understanding of the usefulness of metric spaces in various parts of mathematics

MATHEMATICAL BIOLOGY Mathematical biology is an exciting application of mathematics to biology. In this course students will be introduced to modelling a biological system using ordinary and partial differential equations. Students will learn how to analyse and interpret these models.

Four principal biological topics are considered. These are the Population dynamics, Propagation of Signals in Nerve Cells, Biological Waves, and Morphogenesis (pattern formation) and the Turing Instability. The theories for these problems are well established, but are still the subject of much research. The mathematical methods used will include the use of phase planes, finding travelling wave solutions, and solving reaction-diffusion equations.

MATHEMATICAL ECOLOGY This course aims to teach the application of differential equations and difference equations to problems in ecology. It will provide an understanding of the mathematical modelling methods that can be used to describe population dynamics and epidemiological processes in ecological systems. It will provide training in the mathematical and numerical techniques used to analyse and describe ecological systems.

By means of lectures, tutorials, computer labs, and private study students will be introduced to single species and multi-species difference equation models. Models will be analysed using cobweb diagrams and bifurcation analysis. Examples will be taken from insect population dynamics and harvesting problems. Students will also be introduced to ordinary differential equation models of infectious diseases. We will derive threshold conditions for epidemic outbreaks;calculate the basic reproductive rate of a disease; investigate vaccination strategies to control infection, including pulse vaccination strategies. Finally, we consider the application of delay differential equations to ecology, which will include the derivation of a critical delay for stability in a single delay differential equation and the construction of periodic solutions for piecewise constant negative feedback examples. Lastly, to complement the lectures we will use Matlab to investigate examples taken from the difference equation models and epidemiological models.

MATHEMATICAL METHODS I The course aims to give students a qualitative (existence and uniqueness results without proof) and quantitative (via exact techniques) understanding of the solutions and properties of ordinary differential equations, both linear and non-linear. It also aims to give students an understanding of how some of these methods can be used to solve partial differential equations.

By means of lectures, tutorials and private study students should be able to (a) recognise linear and nonlinear equations; (b) construct Green's functions and handle delta functions for simple systems of equations; (c) construct exact solutions to second order linear differential equations by using a variety of techniques; (d) solve simple eigenvalue/boundary value problems; (e) solve simple partial differential equations by separation of variables.

MATHEMATICAL METHODS II This course continues the sequence of mathematical methods developed in Mathematical Methods I and focuses largely on partial differentia differential equations (PDEs). Commonly occurring PDEs are discussed together with suitable initial and boundary conditions. Various solution techniques are covered such as the Fourier transform, Green's function methods, separation of variables, propagation of waves and the method of characteristics.

By means of lectures, tutorials and private study students should be able to (a) solve a first order linear or quasilinear PDE; (b) determine the regions of the xy-plane in which the general second order PDE in two dimensions is hyperbolic, parabolic and elliptic. Reduce the equation to its canonical form and determine the characteristic equations; (c) reduce the solution of a PDE to finding solutions of ODEs, by separation of variables in polar coordinates and find the particular solution for given initial and/or boundary conditions; (d) find series solutions for second order ODEs about ordinary and regular singular points; (e) deduce properties of special functions (such as orthogonality and recurrence relations) from their governing equations, generating functions or series solutions. Construct systems of orthogonal polynomials using the Gram-Schmidt method; (f) solve PDEs using a range of methods including the Fourier transform and its associated convolution theorem, Green's functions and the method of characteristics.

MATHEMATICAL MODELLING I To provide an introduction to the basic theoretical ideas in the modelling of continuous media. Topics to be discussed are conservation laws in one-dimensional media and their use in formulating equations of motion, the notions of stress and strain and shear, constitutive equations for fluids and solids with applications to fluid mechanics and linear elasticity.

By means of lectures, tutorials and private study students should be able to (a) derive and use various transport formulae in one-dimension; (b) establish the equations of motion for a continuum in one-dimension; (c) understand conservation of mass, momentum and energy; (d) understand the notion of stress, strain and shear; (e) distinguish between constitutive laws for fluids and solids; (f) formulate and solve basic problems in fluid mechanics and linear elasticity involving one-dimensional motion and deformation, for example, the bending of beams, waves on strings, waves in a bar, shock waves in a piston etc; (g) solve simple two and three-dimensional problems with radial symmetry.

WRITING AND PRESENTING MATHEMATICS In this course students develop the skills needed for writing and presenting mathematical material. This is done through project work related to a variety of topics in Pure and Applied Mathematics and by presenting their results both on paper and orally. Assessment is by means of two short projects. Use of the software needed to undertake these projects is taught at the beginning of the course. By the end of this programme students will be able to (a) use the software

LaTeX to prepare and Powerpoint to orally present documents with a significant amount of mathematical and graphical content; (b) express themselves fluently when writing and presenting mathematical material; (c) use the software packages Mathematica and Matlab so as to facilitate the solving of a number of mathematical problems, appropriate to the other courses in their degree programme; (d) demonstrate a better understanding of how use of mathematical software can both give extra insight into mathematical problems and help in solving them.

Appendix 2 - Level 4 Courses

ALGEBRAIC TOPOLOGY The basic idea of the algebraic topology is to find a formal way of translating geometric problems into an appropriate algebraic language. If this is done successfully then the geometric problem is usually reduced to a fairly simple piece of algebra and the problem can be solved by algebraic means. Typical questions and statements in algebraic topology are "The Sandwich-Theorem": no matter how badly you make a sandwich out of two pieces of bread and some filling, it is always possible to find a fair cutting of the sandwich into two pieces; "The Borsuk-Ulam-Theorem": let us associates to each point of the Earths surface two numbers, namely the temperature at that point and the air pressure there. Then there is always at least one point which has the same temperature and air pressure as the diametrically opposite point; "The hairy dog theorem": if you have a dog which is completely covered in hair, then there is no way of combing that hair smooth so that there is no parting or bald spot.

Although these statements sound easy, they are very deep geometric facts about the shapes of dogs, sandwiches and the Earth, especially since they are true for any shape or size of dog, sandwich, or planet... In the course we will develop the algebraic tools to solve these questions. On the other hand we will use geometric tools to show the so-called "Fundamental theorem of algebra" which states that every non-constant complex polynomial $p(z)$ in one variable has some complex root.

Topics to be covered include (a) Homotopy and homotopy equivalence. (b) The fundamental group with examples. (c) The Brouwer fixed point theorem. (d) The fundamental theorem of algebra. (e) Van-Kampen theorem. (f) Covering spaces, path lifting and monodromy theorem. (g) The sandwich theorem. (h) The Borsuk-Ulam theorem. (i) Simplicial homology theory. (j) Eulers formula $v - e + f$. (j) The Lefschetz fixed point theorem.

DIFFERENTIAL GEOMETRY The course introduces fundamental aspects of modern differential geometry and some of its applications to physics (Hamiltonian mechanics) and other branches of mathematics (differential equations and Lie theory). The course aims to introduce the notions of *manifold*, *tangent* and *cotangent* space and *smooth* map based on familiar ideas in $\mathbb{R}^n$; to introduce differential geometric structures on manifolds such as *connections* and *metrics* and to study their geometric properties; to develop *Riemannian* manifolds and *geodesics*; to introduce *Lie* groups and their actions; to apply these ideas to problems drawn from *Hamiltonian mechanics* and *differential equations*.

Topics to be covered include (a) **Calculus on $\mathbb{R}^n$**: The derivative. Maps. Tangent space. (b) **Smooth manifolds**: Charts and altases, differentiable structures, smooth maps, embeddings, immersions, submanifolds of $\mathbb{R}^n$ as abstract manifolds. (c) **Tangent and cotangent spaces**: Idea of a vector. Tangent bundle. One-forms. Cotangent bundles. Multilinear maps between tangent/cotangent spaces. Tensors. (d) **Forms**: Exterior algebra. Exterior derivative. Cartan structure equations. (e) **Connections**: General connections. Connection forms. (f) **Riemannian manifolds**: Metrics. Levi-Civita connection. Geodesics. Holonomy. (g) **Lie groups**: Lie groups and algebras. Matrix groups. Lie derivative. Symmetries of manifolds. Symmetries of differential equations. (h) **Hamiltonian mechanics**: Symplectic manifolds. Hamiltonian functions. Invariants. Symmetries and integrals of motion.

DISCRETE MATHEMATICS Discrete mathematics is the part of mathematics which does not involve notions of continuity (as opposed to calculus and analysis). Most of this course is concerned with properties of the integers. There is also some material on finite sets, including combinatorics, which means counting the elements of finite sets, and graph theory, which involves studying diagrams of finite sets of lines and points. This course also serves as an introduction to number theory.

Students attending the course are expected to acquire a good working knowledge of divisibility and primes, congruences, binomial coefficients, recurrence, the Inclusion-Exclusion Principle, certain work of Fermat and Euler, particular as it relates to RSA cryptography, squares and quadratic reciprocity, and topics in graph theory. By the end of the course students should be able to (a) evaluate multiplicative functions, including the divisor function, the sum of divisors function, and Euler's phi function, (b) solve linear congruences and simultaneous linear congruences. (c) find closed-form expressions of functions given by recurrence relations, (d) apply theorems such as Euler's theorem and Fermat's little theorem to reduce congruences, and (e) compute Legendre symbols using the Law of Quadratic Reciprocity and related theorems.

FINANCIAL MATHEMATICS The course aims to describe the various instruments of Finance, to explain how they work, and to develop the mathematical techniques that are required to make use of these instruments.

Topics to be covered include 1. Jensen's inequality. 2. Simple and compound interest and the time-value of money as embodied in the concept of discounting. The valuation of annuities-certain. The valuation of securities. Cumulative sinking funds. The effect of taxation. 3. The concept of a derivative and the related terminologies such as long, short, arbitrage, hedging, speculation, forward contract, futures contract, close out etc. Concept of an option (American and European) and associated terminologies such as call option, put option etc. The put-call parity condition for European and American options 4. Review of some basic probability including probability density functions, the Normal distribution and the Binomial distribution. 5. The risk-neutral valuation. The valuation of call and put options by lattice methods, by interpreting probability density functions and by solving the Black-Scholes partial differential equation with appropriate boundary conditions and initial/terminal conditions. Concepts of volatility, realised volatility and implied volatility.

GALOIS THEORY Much early algebra centred around the search for explicit formulæ for roots of polynomial equations in one or more unknowns. The solution of linear and quadratic equations in one unknown was well understood in antiquity, while formulæ for the roots of general real cubics and quartics were known solved by the 16th century. Such solutions involved complex numbers rather than just real numbers. By the early 19th century no general solution of a general polynomial equation 'by radicals' (i.e., by repeatedly taking n-th roots for various n) was found despite considerable effort by many outstanding mathematicians. Eventually, the work of Abel and Galois led to a satisfactory framework for fully understanding this problem and the realization that the general polynomial equation of degree at least 5 could not always be solved by radicals. At a more profound level, the algebraic structure of *Galois extensions* is mirrored in the subgroups of their *Galois groups*, and this allows application of group theoretic ideas to the study of fields. This *Galois Correspondence* is a powerful idea which can be generalized to apply to such diverse topics as ring theory, algebraic number theory, algebraic geometry, differential equations and algebraic topology. Because of this, Galois theory in its many manifestations is a central topic in modern mathematics.

This course aims to introduce the basic theory of field extensions and their automorphisms, leading up to the *Galois Correspondence* for *Galois extensions*. Although the main emphasis will be on extensions of the rational numbers $\mathbb{Q}$ and other subfields of the complex numbers $\mathbb{C}$, the techniques also apply to examples such as finite fields. Topics to be covered include (a) Integral domains, fields, polynomial rings and function fields. (b) Fields and their extensions; algebraic extensions; monomorphisms and automorphisms; algebraic closure; separable and normal extensions. (c) Galois extensions; the Galois Correspondence; examples and applications, including finite fields, cyclotomic fields, symmetric functions, solvability and radical extensions.

HAMILTONIAN MECHANICS Hamiltonian Mechanics was invented in the 19th century as a reformulation of classical and Lagrangian mechanics. The formulation is in terms of coupled first order equations rather than the conventional second order Newton's law of motion. This approach does not, in general give new ways to solve the equations analytically but it does have several features that make it significantly different from traditional classical mechanics. Firstly, like Lagrangian mechanics it doesn't require the often tricky analysis of forces in the system; instead one uses the 'Hamiltonian' which is an object that can be obtained from simple energy considerations. Secondly, it gives a geometric approach involving phase plane diagrams and allows qualitative features of a system to be analysed in addition to more traditional quantitative features.

In the course, both Lagrangian and Hamiltonian dynamics will be investigated including many different examples of central force and rigid body problems. We will be looking to solve problems involving particles on curves and surfaces and will be able to deal with constraints using the geometric treatment of Lagrange multipliers. In addition to the above we will use symmetries and geometry to aid our understanding of these problems.

Near the end of the course we will have some time to look at the more modern topic of bifurcations. This is where a small change in a parameter can cause significant changes to the geometric behaviour of a system. We shall introduce normal forms and the theory of Birkhoff to investigate some simple examples.

INTEGRATION Integrals play a central role in mathematics. This course provides a rigorous practical guide to the use of integrals and introduces the beautiful and powerful theory due to Lebesgue. The following topics will be covered.

1. Preliminaries. Integrals of step functions, integrals of continuous functions on compact intervals.

2. Null sets. The class of integrable functions. The monotone and dominated convergence theorems. Integration vs. differentiation.

3. Applications of the theory of integration to Lebesgue spaces, Fourier series and the Fourier transform.

LINEAR ANALYSIS This course is about normed linear spaces, in particular Banach spaces, and the linear mappings between them. The course includes an introduction to Hilbert spaces and also a treatment of Lebesgue integration.

Topics to be covered include (a) **Metric Spaces**: Revision: metric spaces. Interior and Closure: interior of a set, closure of a set, dense sets, separable spaces. Completions: isometric embeddings, completion of a metric space. Compactness: sequential compactness. (b) **Banach Spaces**: Linear Spaces: definitions, examples, linear independence, bases. Normed Spaces: definitions, examples, Minkowski's inequality. Banach Spaces: definitions, examples, finite dimensional case. (c) **Linear Mappings**: Bounded Linear Operators: definitions, examples, continuity and boundedness, operator norm. Dual Spaces: definitions, examples, Hahn-Banach theorem. Operator Algebras: singular and invertible operators, open mapping theorem. (d) **Hilbert Spaces**: Inner Products: definitions, examples, Cauchy-Schwartz inequality, inner product norm, Hilbert spaces, parallelogram law. Orthogonality: orthogonal vectors and sets, orthogonal complements. Orthonormal Sets: definitions, examples, Bessel's inequality, Fourier series, Parseval's inequality, Gram-Schmidt process. Functionals and Adjoints: Riesz representation theorem, definitions only of adjoint, Hermitian, normal, unitary, orthogonal projection. (e) **Lebesgue Integration**: Preliminaries: measure of an open set, null sets, almost everywhere. Step Functions: definition, examples, Lebesgue integral of a step function. The Lebesgue Integral: definitions, relationship between Lebesgue and Riemann integrals. Convergence Theorems: monotone convergence theorem, dominated convergence theorem, examples. Measurable Functions: outline only.

MATHEMATICAL BIOLOGY See details in the Level 3 Courses Appendix.

MATHEMATICAL ECOLOGY See details in the Level 3 Courses Appendix.

MATHEMATICAL MODELLING II This course introduces the fundamental concepts, principles and mathematical theory of Continuum Mechanics, which describes the material properties, deformation and motion of solids and fluids over all scales except those where quantum effects are important. It provides the background for understanding current research in e.g. smart materials, soft tissue mechanics, and microfluidics. Topics include:

1. Introduction to the theory of 3D cartesian tensors, eigenvalues and eigenvectors of a symmetric second-order tensor, and the tensor product.

2. Kinematics of deformation and motion, the material time derivative, 2D deformations and flows, simple shear, shearing motion, incompressibility). Eulerian and Lagrangian descriptions of motion, transport formulae.

3. Conservation laws of linear and angular momentum, the Cauchy stress tensor, equations of motion and equilibrium, energy balance.

4. Constitutive laws and objectivity, elasticity and viscous fluids, Navier-Stokes equations and plane parallel flows.

5. Material symmetry, isotropy and anisotropy.

6. Hyperelasticity and the strain-energy function.

7. Boundary-value problems.

8. Exact solutions in nonlinear elasticity.

MATHEMATICAL MODELLING III This course represents a natural continuation of the material covered in Mathematical Modelling 2. While the general continuum theories developed there provide much of the theoretical framework needed to understand the physics of complex phenomena, in general, the resulting mathematical models are complicated and their solutions are accessible only through numerical means. It is the purpose of this course to show how, by a judiciously chosen number of simplifying hypotheses, it is possible to derive a consistent linear theory that leads to boundary-value problems which can be easily understood with the techniques developed in earlier courses such as Mathematical Methods I/II. The bending and torsion of three-dimensional bars, the propagation of waves in solid materials, and the behaviour of clays and gels are but a few representative applications that will be discussed. Topics to be covered include some of the following: (a) Linearized kinematics (infinitesimal strains and infinitesimal rotations). Linear elasticity: isotropic and anisotropic materials. The generalized Hooke's Law. Elastic moduli. (b) Typical applications of linear elasticity: plane stress, plane strain, anti-plane strain, Airy stress functions. (c) Existence, uniqueness, well-posedness, Betti's theorem, Saint-Venant's principle, minimum energy principle, maximum complementary energy and mean value theorem. (d) Elastodynamics: wave propagation; waves in isotropic and anisotropic materials, Rayleigh waves, reflections from a boundary, waves in layered media, Love waves. (e) Introduction to viscoelasticity.

NONLINEAR WAVES One of the earliest recorded observations of a nonlinear wave was by John Scott Russell, a Scottish engineer back in 1834. Whilst riding his horse besides the Union Canal he observed a boat stop suddenly, the water around the prow of the boat accumulated into a large 'wave' and rolled forward along the canal. He followed this wave for several miles on horse back until he eventually lost it in the windings of the canal. It was many years later in 1895 that Korteweg and de Vries derived an equation for surface waves, to explain this phenomena. This is just one example of a nonlinear wave. Nonlinear wave equations and their solutions have wide applications in various branches of applied mathematics and physics. In this course we shall examine some of the nonlinear differential equations amenable to analytical solution. There are a number of different techniques used to solve these equations, we shall look at some of these methods and the properties of the resulting solutions.

Topics to be covered are (a) The method of characteristics; (b) The kinematics wave equation and shocks. (c) Applications to traffic flow. (d) Burgers' equation including applications of the Fourier transform. (e) Solitons and Hirota's method.

NUMBER THEORY This course builds on the basic ideas already met in earlier years. Number theory is an old subject that has provided some of the basic stimuli or modern algebra, and difficult problems such as proving Fermat's Last Theorem (now known as Wiles' Theorem) and the Goldbach twin primes conjecture continue to provide easily explained challenges which require powerful techniques to make substantial progress. Applications to cryptography and error correction also make number theory important as applicable mathematics.

Topics covered will include material on the following: congruences (including Fermat's little theorem, Euler's theorem and quadratic reciprocity); diophantine problems; continued fractions and Pell's equation; basic ideas of algebraic number theory (including quadratic and cyclotomic number fields, rings of integers, factorisation theory); analytic number theory.

NUMERICAL ANALYSIS Numerical analysis is the study of algorithms and approximations for the solution of mathematical problems that may be otherwise intractable or whose analytic solution is not useful for one reason or another. Examples include the solution of equations of the form $f(x) = 0$, the solution of linear systems $Ax = b$ and associated factorisation of matrices, the approximation of a function by a polynomial, numerical integration and the numerical solution of ordinary differential equations. We use the results of real analysis to generate, analyse and apply algorithms for the solution of such problems.

As an example, the following iterative method gives rational approximations to $\sqrt{2}$,

$$x_{n+1} = x_n \frac{x_n^2 + 6}{3x_n^2 + 2}.$$

The method converges very quickly and involves only basic mathematical operations. With an initial guess of $x_0 = 3/2$ the error is around 10^{-40} after three iterations. To achieve the same accuracy with interval bisection would require over 100 iterations. The reasons for this will be studied in the course.

NUMERICAL SOLUTIONS OF PARTIAL DIFFERENTIAL EQUATIONS Partial differential equations lie at the heart of applied mathematics, arising in applications as varied as weather/climate prediction, elasticity, biology, and economics. However, unlike the equations encountered in courses on ODEs or PDEs, the equations governing 'real-world' applications rarely have closed-form solutions. One thus has to resort to finding approximate numerical solutions.

Topics to be covered are (a) **Introduction**: Why develop numerical techniques at all? Review of classification of PDEs: differences in equations and associated boundary conditions means different numerical techniques will be needed. Derivation of basic finite difference formulae. (b) **Finite differences in 1 Dimension**: Boundary Value Problems: Tridiagonal algorithm, and numerical implementation thereof. Implementation of Dirichlet and Neumann boundary conditions. Accuracy, and improved accuracy via Richardson extrapolation. (c) **Elliptic Equations in 2 or 3 space Dimensions**: Jacobi and Gauss-Seidel iteration, and convergence thereof. Nonlinearities. Irregular grids. (d) **Parabolic Equations in 1 Space Dimension**; Time-stepping via Euler and Crank-Nicolson methods. Consistency, Convergence. Stability via the Fourier and Matrix Methods. Nonlinearities. (e) **Parabolic Equations in 2 or 3 Space Dimensions**: Advantages/disadvantages of Euler, Crank-Nicolson, and ADI (alternating direction implicit) methods. (f) **Eigenvalue Problems**: Inverse iteration, complex eigenvalues.