University of Huddersfield
School of Computing and Engineering

CFM2103 Mathematical Programming

Practical **Week 7**

Work through the questions included below. If you get stuck, please revisit the information on the slides from the previous weeks before you ask for help.

In all the questions included below you must make use of what you have learned about `strings` to neatly display the output of your code on the computer screen.

---

1. There are many ways to find numerical approximations for the mathematical constant π. For example,

$$\frac{\pi}{8} \simeq \sum_{k=0}^{N} \frac{1}{(4k+1)(4k+3)} \qquad \text{or} \qquad \frac{\pi^2}{6} \simeq \sum_{k=1}^{N} \frac{1}{k^2}\,,$$

where $N \gg 1$ is a natural number; the larger N is, the better the approximation of the expressions on the left-hand side of these formulae. For each of these formulae, write a Python function that takes N as input from the user, and displays on the screen the corresponding approximation, the *absolute* error, as well as the *relative* error. Use these functions in suitable for-loops to display on the screen the above information for $N = 5,\ 10,\ 15,\ \ldots,\ 95,\ 100$. Which formula is better?

[Hint: Sums are always calculated by using some sort of *accumulator pattern*. Please check out the solved examples for this week (available on Brightspace) – the first example is very similar to this question.]

2. Consider the function $f(x) = x^3 - 6x^2 + 3x + 10$, with $-2 \leq x \leq 6$. Use the *finite-difference* approximation for the derivative of this function,

$$f'(x) \simeq \frac{f(x+h) - f(x-h)}{2h}\,,$$

in order to identify its stationary points.

[Hint: You can plot the *numerical approximation* of $f'(x)$ and find the stationary points by visual inspection – these are the points where the graph of the derivative crosses the horizontal axis. You may also find it useful to look at the second solved example on Brightspace.]

3. Use the trapezium and Simpson's methods to find approximations for the integrals:

$$(a)\ \int_0^2 \ln(1+e^x)\,dx\,; \qquad (b)\ \int_0^\pi \frac{dx}{\sqrt{x^4+1}}\,,$$

accurate to within six significant figures.

[Hint: You have the implementation for the above methods on Brightspace.]

4. The *logarithmic integral* is a special mathematical function defined by the equation

$$\mathrm{li}(x) = \int_2^x \frac{1}{\ln t}\,dt\,.$$

For large x, the number of prime integers less than or equal to x is closely approximated by $\mathrm{li}(x)$. For example, there are 46 primes less than 200, and li(200) is around 50. Find li(200) with three signficant figures by means of the trapezium and Simpson's methods.
Approximate li(700) using the same numerical methods, and compare the result with the actual number of primes less than or equal to 700 (you already have the code for finding those primes).

[Hint: same as for the previous question.]

Optional question:
If you would like to receive additional feedback on your work, you should attempt the questions included below and submit your computer code in a zipped folder on Brightspace by no later than 5:00 PM next Tuesday.

Consider the function

$$f(x) = \mathrm{e}^{-ax}\sin(bx)\,, \qquad 0 \le x \le 20\,,$$

where a, $b > 0$ are given constants.

1. Write two functions that calculate the numerical approximations of the first- and second-order derivatives based on the central finite-difference formulae,

$$f'(x) \simeq \frac{f(x+h) - f(x-h)}{2h}\,, \qquad f''(x) \simeq \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}\,,$$

   where $0 < |h| \ll 1$ and $0 < x < 20$. The constants a and b should be treated as user inputs.

2. Create separate plots for $a = 3$, $b = 2$, in which you show both your numerical approximations and the exact results (one plot for the first-order derivative and another one for the second-order derivative). Add axes labels and use a legend to clearly indicate the curves in your plots.